

3E1486

Roll No. : _____

Total Printed Pages : **4**

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B. Tech. (Sem. III) (Main/Back) Examination, February - 2010
(Common for 3EE6.1 & 3EX1)
(Mathematics)

Time : 3 Hours]

[Total Marks : 80

[Min. Passing Marks : 24

Attempt overall five questions in all. Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitable be assumed and stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. _____ Nil _____

2. _____ Nil _____

1 (a) Find the Laplace transform of the following functions

(i) $e^{-2t} \cos^2 t$ (ii) $t^2 e^t \sin t$

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(b) Using Laplace transform solve $\frac{d^2 x}{dt^2} + \frac{dx}{dt} = 2$. Given that $x = 3$

at $t = 0$ and $\frac{dx}{dt} = 1$ at $t = 0$.

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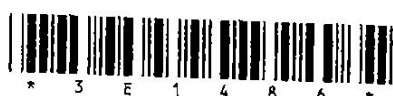
OR

1 (a) Find the Laplace transform of $\sin \sqrt{x}$. Hence show that

$$L \left\{ \frac{\cos \sqrt{x}}{\sqrt{x}} \right\} = \left(\frac{\pi}{s} \right)^{1/2} e^{-1/4s}$$

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[Contd...

(b) Apply convolution theorem to evaluate $L^{-1} \left\{ \frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right\}$.

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2 (a) Find the Fourier sine transform of $f(x) = e^{-x}$, $x \geq 0$. Also

show that $\int_0^{\infty} \frac{x \sin mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}$, $m > 0$.

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(b) Solve the differential equation by using Fourier sine transform

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0 \text{ subject to the condition}$$

(a) $u(0, t) = 0$

(b) $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ when $t = 0$ and

(c) $u(x, t)$ is bounded.

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OR

2 (a) Find the Fourier sine and cosine transforms of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$$

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(b) Solve the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$, $t > 0$, subject

to $u(x, 0) = f(x)$ where $f(x) = \begin{cases} u_0, & |x| < a \\ 0, & |x| > a \end{cases}$.

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- 3 (a) Expand $f(x) = |\cos x|$ in a Fourier series in the interval $(-\pi, \pi)$.

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- (b) The turning moment T units of the crank shaft of a steam engine is given for a series of values of the crank angle θ in degrees :

θ°	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Find the first four terms in a series of sine of represent T . Also calculate T when $\theta = 75^\circ$.

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OR

- 3 (a) Find the Fourier series to represent the function $f(x) = x^2$, $-\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{8} = 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \dots$

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- (b) Find the equation of the curves for which the functional $\int_0^1 \{(y')^2 + 12xy\} dx$, $y' = \frac{dy}{dx}$ with $y(0) = 0$ and $y(1) = 1$ can be extremised.

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- 4 (a) Show that the function $u + iv = f(z)$, where

$$f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Satisfy the Cauchy-Riemann equations at the origin, yet $f'(0)$ does not exist.

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- (b) Evaluate the following integral by using Cauchy's integral

formula $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$.

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- 4 (a) If $f(z) = u + iv$ be an analytic function of z and

$$u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x} \quad \text{find } f(z).$$

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- (b) Show that the transformation $W = z + \frac{a^2 - b^2}{4z}$ transforms, the circle of radius $\frac{1}{2}(a+b)$, centre at the origin in the z -plane into ellipse of semi-axes a, b in the W -plane.

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- 5 (a) State and prove Taylor's series.

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- (b) Evaluate the following integral by Contour

integration : $\int_0^{\infty} \frac{dx}{1+x^4}$.

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OR

- 5 (a) Expand $\frac{1}{z(z^2 - 3z + 2)}$ in Laurent series for the region :

(i) $0 < |z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| < 2$

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- (b) Evaluate $\oint_C \frac{z \cos z}{\left(z - \frac{\pi}{2}\right)^2} dz$ where C is the circle $|z-1|=1$.

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