3E1486

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B.Tech. (Sem. III) (Main/Back) Examination, February - 2010
(Common for 3EE6.1 & 3EX1)
(Mathematics)

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

*Attempt overall **five** questions in all. Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitable be assumed and stated clearly.*

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. Nil

1 (a) Find the Laplace transform of the following functions

(i) $e^{-2t}\cos^2 t$ (ii) $t^2 e^t \sin t$

8

(b) Using Laplace transform solve $\frac{d^2x}{dt^2}+\frac{dx}{dt}=2$. Given that $x=3$ at $t=0$ and $\frac{dx}{dt}=1$ at $t=0$.

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OR

1 (a) Find the Laplace transform of $\sin\sqrt{x}$. Hence show that

$$L\left\{\frac{\cos\sqrt{x}}{\sqrt{x}}\right\}=\left(\frac{\pi}{s}\right)^{1/2} e^{-1/4s}.$$

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(b) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$.

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2 (a) Find the Fourier sine transform of $f(x)=e^{-x}$, $x \geq 0$. Also show that $\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}$, $m>0$.

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(b) Solve the differential equation by using Fourier sine transform

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $x>0$, $t>0$ subject to the condition

(a) $u(0, t)=0$

(b) $u(x,0)=\begin{cases} 1, & 0<x<1 \\ 0, & x \geq 1 \end{cases}$ *when* $t=0$ and

(c) $u(x, t)$ is bounded.

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OR

2 (a) Find the Fourier sine and cosine transforms of

$$f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x \geq 2 \end{cases}$$

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(b) Solve the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$ $t>0$, subject to $u(x,0)=f(x)$ where $f(x)=\begin{cases} u_0, & |x|<a \\ 0, & |x|>a \end{cases}$.

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3 (a) Expand $f(x) = |\cos x|$ in a Fourier series in the interval $(-\pi, \pi)$.

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(b) The turning moment T units of the crank shaft of a steam engine is given for a series of values of the crank angle θ in degrees :

$\theta°$	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Find the first four terms in a series of sine of represent T. Also calculate T when $\theta = 75°$.

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OR

3 (a) Find the Fourier series to represent the function $f(x) = x^2$, $-\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{8} = 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots\ldots$.

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(b) Find the equation of the curves for which the functional $\int_0^1 \left\{(y')^2 + 12\,xy\right\} dx$, $y' = \frac{dy}{dx}$ with $y(0) = 0$ and $y(1) = 1$ can be extremised.

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4 (a) Show that the function $u + iv = f(z)$, where

$$f(z) = \begin{cases} \frac{x^3(1+1') - y^3(1-1')}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Satisfy the Cauchy-Riemann equations at the origin, yet $f'(0)$ does not exist.

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(b) Evaluate the following integral by using Cauchy's integral formula $\oint_C \frac{e^{2z}}{(z+1)^4}\, dz$ where C is the circle $|z| = 2$.

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OR

4 (a) If $f(z)=u+iv$ be an analytic function of z and

$$u+v=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$$ find $f(z)$.

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(b) Show that the transformation $W=z+\frac{a^2-b^2}{4z}$ transforms, the circle of radius $\frac{1}{2}(a+b)$, centre at the origin in the z-plane into ellipse of semi-axes a, b in the W-plane.

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5 (a) State and prove Taylor's series.

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(b) Evaluate the following integral by Contour integration : $\int_0^\infty \frac{dx}{1+x^4}$.

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OR

5 (a) Expand $\frac{1}{z(z^2-3z+2)}$ in Laurent series for the region :

(i) $0<|z|<1$

(ii) $1<|z|<2$

(iii) $|z|<2$

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(b) Evaluate $\oint_C \frac{z\cos z}{\left(z-\frac{\pi}{2}\right)^2}dz$ where C is the circle $|z-1|=1$.

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