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4E2989

B. Tech. (Sem. IV) (Main / Back) Examination, June/July - 2013
Electronics & Comm.
4EC5 Random Variables & Stochastic Processes

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

Attempt any five questions, selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.
Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

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UNIT - I

1 (a) Consider "a" apples are distributed randomly among "b" children. Determine the probability that a particular child receives "c" apples; where c < a. **6**

(b) Suppose urn 1 contains "p" white balls and "q" black balls, and urn 2 contains "r" white balls and "s" black balls. One ball of unknown color is transferred from the first urn into the second urn and then a ball is drawn from the latter. Determine the probability that it will be a white ball. **10**

OR

1 Prove following :

(a) If $P(A/B)$ is the conditional probability of A given B, then show that

(i) $P(A/B) \geq 0$ **4**

(ii) $P(S/B) = 1$ **4**

(iii) $P(A_1 \cup A_2 / B) = P(A_1/B) + P(A_2/B)$ if $A_1 \cap A_2 = \phi$ **4**

(b) If $P(A/B) > P(A)$, then prove that $P(B/A) > P(B)$.
Note : In (ii) of (a), S denotes the universal sample space. **4**

UNIT - II

2 (a) Determine the mean and variance of the uniform distribution $U(a,b)$.

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(b) Consider the sequence of bernoulli trials with probability p of success. This sequence is observed until the first success occurs. Let the random variable X represent the trial number on which the first success occurs. Then the pmf is given by

$$p_X(x) = P(X = x) = (1-p)^{x-1}\,p;\ x = 0,1,2,\ldots\ldots \to (i)$$

because there must be $x-1$ failures before the first success occurs on trial x. The random variable X defined in this problem is called as "geometric random variable" with probability p.

(i) Show that $p_X(x)$ given in above equation

(a) Satisfies the expression

$$\sum_k p_X(x_k) = 1$$

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(ii) Determine the cdf $F_X(x)$ of X.

5

OR

2 (a) If the density function of a continuous random variable is $f(x) = Ce^{-b(x-a)};\ a \le x$.

where a,b,c are real constant. Show that $b = c = \frac{1}{\sigma_X}$ and $a = \mu_X - \sigma_X$. Where $\mu_X = E[X]$ and $\sigma_X^2 = \text{var}[X]$.

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(b) State and explain the "memory less property" of exponential distribution.

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UNIT - III

3 (a) The joint pdf of a bivariate random variable (X, Y) is given by

$$f_{XY}(x,y)=\begin{cases}\frac{kx}{y} ; 1<x<2 ; 1<y<2 \\ 0 ; \text{ otherwise}\end{cases}$$

where k is the constant.

(i) Determine K. 5

(ii) are X and Y independent 5

(b) Varify the property :

$$P(x_1 < X \le x_2, Y \le y) = F_{XY}(x_2, y) - F_{XY}(x_1, y)$$

6

OR

3 (a) Varify the property :

$$P(X \le x, y_1 < Y \le y_2) = F_{XY}(x, y_2) - F_{XY}(x, y_1)$$

6

(b) A joint pdf of a bivariate random variable (X,Y) is given by

$$f_{XY}(x,y)=\begin{cases}\frac{1}{y}e^{-x/y}e^{-y} ; & x>0, y>0 \\ 0 & ; \text{ otherwise}\end{cases}$$

(i) Show that $f_{XY}(x,y)$ satisfies the equation

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1$$

5

(ii) determine $P(X > 1 / Y = y)$ 5

UNIT - IV

4 (a) Consider a continuous random variable X, prove that

$$E(X)=\int_{0}^{\infty}\left[1-F_X(x)\right]dx-\int_{-\infty}^{0}F_X(x)\,dx$$

where $E(X)$ denotes the expectation of random variable X. 8

(b) For the correlation - coefficient of X and Y (defined as ρ_{XY}), prove that

$$|\rho_{XY}| \leq 1$$

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(c) Consider the random variable X, which takes the values $x_1 = -2$ and $x_2 = 2$ with pmf's $p_X(x_1) = p_X(x_2) = \frac{1}{2}$. Find the characteristic function of X.

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OR

4 State and explain the "central limit theorem in detail with appropriate mathematical equations.

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UNIT - V

5 (a) The psd of white noise $(N_0/2)$ is 6×10^{-6} watts/Hz, is applied to an ideal low pass filter with power transfer function 1 and bandwidth = W rad/sec. Find the W so that output average noise power is 15 watts.

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(b) The psd of a zero mean WSS process $X(t)$ is given by

$$S_{XX}(\omega) = \begin{cases} 1 \ ; & |\omega| \leq \omega_0 \\ 0 \ ; & \text{anywhere} \end{cases}$$

Determine whether $X(t)$ and $X\left(t + \frac{\pi}{\omega_0}\right)$ are uncorrelated.

8

OR

5 (a) The power spectrum of Noise $N(t)$ is defined as

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2} \ ; & -W \leq \omega \leq W \\ 0 \ ; & \text{elsewhere} \end{cases}$$

Determine the auto correlation function of $N(t)$ and draw the plot of $R_{NN}(\tau)$ versus τ.

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(b) Explain the "Narrow Banel Gaussian Processes" with appropriate mathematical expressions.

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