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4E2989

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B. Tech. (Sem. IV) (Main / Back) Examination, June/July - 2011
Electronics & Communication
4EC5 Random Variables & Stochastic Processes

Time : **3** Hours] [Total Marks : **80**
[Min. Passing Marks : **24**

Attempt overall ***five*** *questions selecting* ***one*** *question from each unit. All questions are carry* ***equal*** *marks.*
(Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly).
Units of quantities used/calculated must be stated clearly

Use of following supporting material is permitted during examination.
(Mentioned in form No. 205)

1. Nil 2. Nil

UNIT - I

1 (a) Show that for any events A and B in S
$$P(B)=P(B/A)P(A)+P(B/\overline{A})P(\overline{A})$$
4

(b) Let A and B be events in a sample space S. Show that if A and B are independent, then show that :

(i) A and $\overline{B}$ are independent 4

(ii) $\overline{A}$ and B are independent 4

(iii) $\overline{A}$ and $\overline{B}$ are independent. 4

OR

1 (a) In a system, there are n-components connected in series. This system works successfully. The operation of each component is independent to each other. The probability of successful operation of the components is $p_i; i=1,2,3,\dots\dots n$. Find the probability that the system functions successfully. The corresponding diagram is shown in Fig. 1 [a]

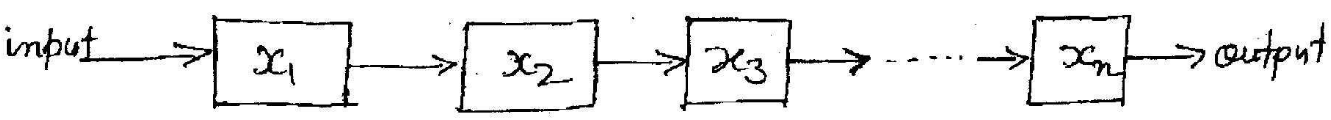

Fig. 1

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(b) Let the telegraph source is generating two symbols, dots and dashes. It is observed that the dots were twice as likely to occur as dashes. Determine the probabilities of the dot's occurring and the dash's occurring.

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UNIT - II

2 (a) What is random variable ? Explain all the types of random variables with suitable example.

2+6=8

(b) Show that the pdf of normal random variable X satisfies the expression $\int_{-\infty}^{\infty} f_X(x)dx = 1$.

i.e. This is legitimate distribution.

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OR

2 (a) Give the applications of the Poisson's random variables.

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(b) Prove the reproductive property of independent Poisson's random variable. Hence find the probability of 5 or more telephone calls arriving in a 9 min. period in a college switch - board, if the telephone calls that are received at the rate of 2 every 3 min. follows a Poisson distribution.

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UNIT - III

3 (a) If $A = \{X \le x\}$ and $B = \{Y \le y\}$ are statistical independent events, then, what shall be the effects on the following :

(i) Joint probability distribution function $F_{XY}(x, y)$

2

(ii) Joint probability density function $f_{xy}(x, y)$

2

(iii) Conditional distribution

2

(iv) Conditional density.

2

(b) The joint pdf of a bivariate RV (X, Y) is given as

$$f_{XY}(x, y) = \begin{cases} \dfrac{Kx}{y} & ; \quad 1 < x < 2, \quad 1 < y < 2 \\ 0 & ; \quad otherwise \end{cases}$$

where K is a constant

(i) determine K

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(ii) are X and Y independent ?

4

OR

3 (a) Explain all the properties of joint pdf and joint cdf

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(b) If the joint pdf of (X, Y) is given by $f_{XY}(x,y) = x + y; 0 \le x, y \le 1$, find the pdf of $U = XY$.

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UNIT - IV

4 (a) State and prove the "Chebyshev inequality"

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(b) Consider X and Y are two random variables where they are defined as

$X = \cos\psi$ and $Y = \sin\psi$

where ψ is another random variable uniformaly distributed over $(0, 2\pi)$. Show that X and Y are uncorrelated.

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OR

4 (a) State and prove all the properties of moment generating function.

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(b) Consider a random variable X which is uniformly distributed over $\left(-\sqrt{4}, \sqrt{4}\right)$. Calculate $P\left\{|X - \mu_X| \ge \frac{3\sigma_X}{2}\right\}$ and then compare it with the upper bound obtained by chebyshev inequality.

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UNIT - V

5 (a) A stationary random process has an autocorrelation function of
$$R_{XX}(\tau) = \begin{cases} 10\left\{1 - \frac{|\tau|}{0.05}\right. & ; \quad |\tau| \le 0.05 \\ 0 & ; \quad elsewhere \end{cases}$$
Determine the mean and variance of the process X(t).

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(b) The psd of N(t) is defined in Fig. 5(b), find out the :

(i) autocorrelation function of N(t).

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(ii) the average power of N(t).

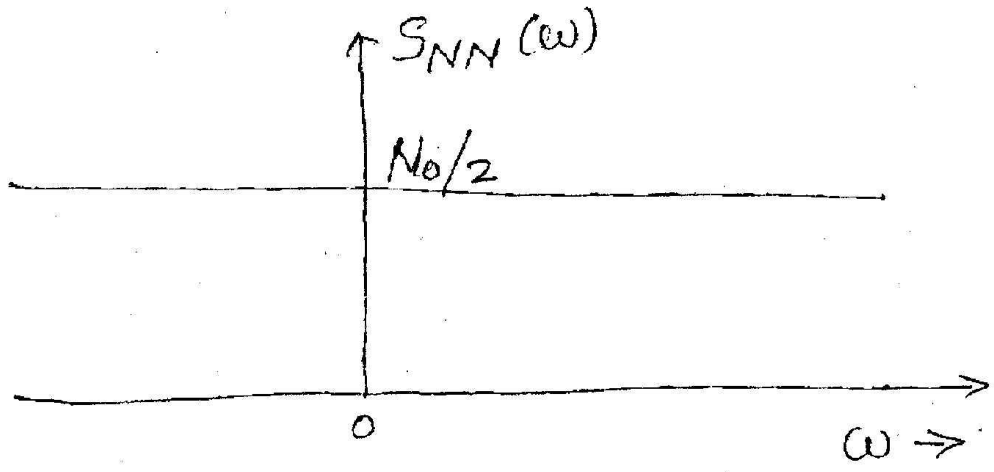

Fig. 5(b)

OR

5 (a) In the fig. 5 (a), X(t) be a input voltage to the circuit and Y(t) be the output voltage. The process X(t) is a stationary random process with zero mean and its autocorrelation function is $R_{XX}(\tau) = e^{-\alpha|\tau|}$.

Determine :

(i) $E[Y(t)]$ 3

(ii) $S_{YY}(w)$ 3

(iii) $R_{YY}(\tau)$ 4

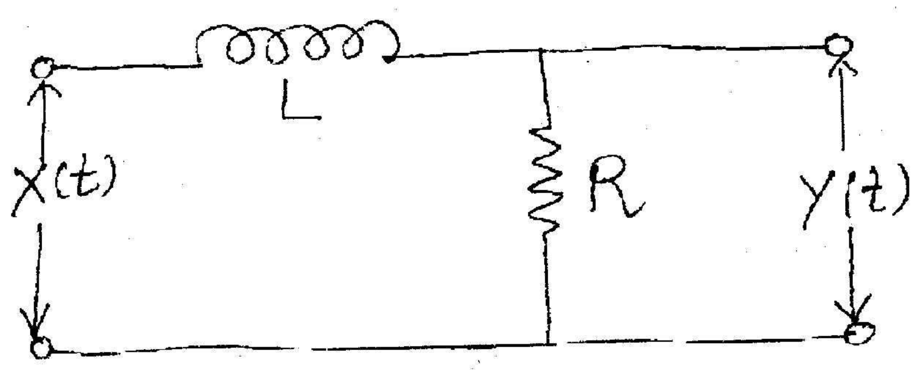

Fig. 5(a)

(b) If Y(t) be the output of an LTI system with impulse response h(t) when a WSS random-process X(t) is applied as input. Show that :

(i) $S_{XY}(w) = H(w)S_{XX}(w)$ 3

(ii) $S_{YY}(w) = H^*(w)S_{XY}(w)$ 3