37	Roll No. :	Total Printed Pages: 4
4E21	B. Tech. (Sem. IV) (Main) Examina Petroleum 4PE1 Mathematics - IV (Com	
Γime :	3 Hours]	[Total Marks : 80 [Min. Passing Marks : 24

rom each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.

Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

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UNIT - I

Find the missing term in the following table 1

$$x \rightarrow 0 \quad 1 \quad 2 \quad 3 \quad 4$$

 $y \rightarrow 1 \quad 3 \quad 9 \quad - \quad 81$

4

Find the form of the function given by the following table (b)

$$x \rightarrow 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

 $y \rightarrow -5 \quad 1 \quad 9 \quad 25 \quad 55 \quad 105$

4

The area A of a circle of diameter d is given for the following (c) values

$$d \rightarrow 80 \quad 85 \quad 90 \quad 95 \quad 100$$

 $A \rightarrow 5026 \quad 5674 \quad 6362 \quad 7088 \quad 7854$

Calculate the area of circle of diameter 105.

8

OR

1 (a) Using Stirling formula to find f(35) from the table

$$x \rightarrow 20 \quad 30 \quad 40 \quad 50$$

$$y \rightarrow 512 \ 439 \ 346 \ 243$$

(b) Using Lagrange's interpolation formula, find the value of y for x=9.5 from table

$$x \rightarrow 7 \quad 8 \quad 9 \quad 10$$

$$y \rightarrow 3 \quad 1 \quad 1 \quad 9$$

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UNIT - II

2 (a) Find f'(1.5) using following data

$$x \rightarrow 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5 \quad 4$$

$$y \rightarrow 3.375 \quad 7 \quad 13.625 \quad 24 \quad 38.875 \quad 59$$

- 8
- (b) Using Euler's modified method, obtain a solution of $\frac{dy}{dx} = x + |\sqrt{y}|$, y(0) = 1 for the range $0 \le x \le 0.4$ in steps of 0.2.

8

OR

2 (a) Using Simpson's " $\frac{1}{3}$ " rule, integrate, $\int_{1}^{1.04} f(x)dx$ from following data

$$x \rightarrow 1$$
 1.01 1.02 1.03 1.04 $f(x) \rightarrow 3.953$ 4.066 4.182 4.300 4.421

8

(b) Apply fourth order Runge-Kutta method to

$$\frac{dy}{dx} = 3x + \frac{1}{2}y, y(0) = 1$$

to determine y(0.1) correct to four decimal places.

UNIT - III

- Derive the result of generating function for Bessel function 3 $J_n(x)$.
 - 8
 - For Legendre's function, show that
 - (i) $P_{n}(1) = 1$
 - (ii) $P_n(-x) = (-1)^n P_n(x)$

8

OR

For Bessel function, show that 3 (a)

$$\left[J_{-\frac{1}{2}}(x) \right]^{2} + \left[J_{\frac{1}{2}}(x) \right]^{2} = \frac{2}{\pi x}$$

State and prove orthogonal properties of Legendre's function. (b)

8

UNIT - IV

4 There are 3 true coins and 1 false coin with 'head' on both (a) sides. A coin is chosen at random and tossed 4 times. If 'head' occurs all the 4 times, what is the probability that the false coin has been chosen and used?

8

Fit a binomial distribution for given data: (b)

$$x \rightarrow 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

 $f \rightarrow \quad 5 \quad 18 \quad 28 \quad 12 \quad 7 \quad 6 \quad 4$

8

OR

Two cards are drawn at random with replacement from a box (a) 4 which contains 4 cards numbered 1, 1, 2 and 2. If X denotes the sum of the numbers shown on the two cards, find the expected value of X.

Compute the coefficient of correlation between x and y, using (b) the following data:

$$x \rightarrow 1 \quad 3 \quad 5 \quad 7 \quad 8 \quad 10$$

 $y \rightarrow 8 \quad 12 \quad 15 \quad 17 \quad 18 \quad 20$

5 (a) Find the extremals of functional

$$\int_{0}^{\pi/2} \left[(y'')^{2} - (y)^{2} + (x)^{2} \right] dx;$$

given that y(0) = 1, y'(0) = 0.

$$y(\pi/2) = 0$$
, $y'(\pi/2) = -1$

8

(b) Find the curve through two points (x_1, y_1) and (x_2, y_2) which when rotated about the x axis, gives minimum surface area.

8

OR

5 (a) Derive Euler's equation.

8

(b) Find the extremals of functional

$$\int_{0}^{\pi/2} \left[(x')^{2} + (z')^{2} + 2yz \right] dx;$$

given that
$$y(0) = 0$$
, $y(\pi/2) = 1$
 $z(0) = 0$, $z(\pi/2) = -1$