3E1626

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B.Tech. (Sem.III) (Main/Back) Examination, 2015
Civil Engineering
3CE6 Advanced Engineering Mathematics

Time : 3 Hours

Total Marks : 80
Min. Passing Marks : 26

Instructions to Candidates :
Attempt any five questions, selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

UNIT - I

1. (a) Find the half-range sine series for $f(x) = x(\pi - x)$, in $0 < x < \pi$ and hence find the sum of the series

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots$$ **(8)**

(b) Obtain the constant term and the coefficient of the first sine and cosine terms in the Fourier series that represents y as given in the table :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(8)

OR

1. (a) *Find the Z-transform of :*

$$u_n = \begin{cases} 5^n , & n < 0 \\ 3^n , & n \ge 0 \end{cases}$$ **(8)**

(b) Find inverse Z-transform of :

$$\bar{u}(z) = \frac{3z^2 + 2z}{(5z-1)(5z+2)}$$ **(8)**

UNIT – II

2. (a) Find the Laplace transform of $\sin\sqrt{t}$. Hence show that

$$L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right) = \left(\frac{\pi}{s}\right)^{1/2} e^{-\frac{1}{4s}}$$ **(8)**

(b) Use Laplace transforms technique to solve :

$(D^2 - 3D + 2)x = 1 - e^{2t}$; given $x(0) = 1$, $x'(0) = 0$, $D = \frac{d}{dt}$ **(8)**

OR

2. (a) Use convolution theorem to evaluate

$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ **(8)**

(b) Use Laplace transform theory to solve

$\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2}$; given $u(x, 0) = \cos 5x$

$u_x(0, t) = 0$, $u\left(\frac{\pi}{2}, t\right) = 0$ **(8)**

UNIT - III

3. (a) Find the Fourier sine transform of $f(x) = e^{-x}$, $x \ge 0$

Also, show that $\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}$, $m > 0$ **(8)**

(b) Solve the integral equation

$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}$. Hence evaluate

$\int_0^\infty \frac{\cos \lambda x}{1+x^2} dx = \frac{\pi}{2} e^{-\lambda}$ **(8)**

OR

3. (a) Find the fourier transform of

$f(x) = \begin{cases} 1, & |x| \le a, \\ 0, & |x| > a \end{cases}$

Also evaluate $\int_{-\infty}^{\infty} \frac{\sin \lambda a \cos \lambda x}{\lambda} d\lambda$ **(8)**

(b) Using Fourier cosine transform, solve

$\frac{\partial \theta}{\partial t} = C^2 \frac{\partial^2 \theta}{\partial x^2}$ subject to the conditions :

(i) $\theta = 0$ when $t = 0$, $x \ge 0$;

(ii) $\frac{\partial \theta}{\partial x} = -\mu$, a constant, when $x = 0$ and $t > 0$.

Assume that $\theta(x,t)$ and $\left(\frac{\partial \theta}{\partial x}\right)$ both tend to zero as $x \to \infty$. **(8)**

UNIT – IV

4. (a) The area A of a circle of diameter d is given for the following values :

d:	80	85	90	95	100
A:	5026	5674	6362	7088	7854

Find approximate area of a circle with diameter 82. (8)

(b) Use Lagrange's formula to interpolate the value of y at x = 10 from the following date :

x:	5	6	9	11
y:	12	13	14	16

(8)

OR

4. (a) Prove that :

$$u_0 + \frac{xu_1}{1!} + \frac{x^2u_2}{2!} + \frac{x^3u_3}{3!} + \ldots = e^x\left[u_0 + \frac{x\Delta u_0}{1!} + \frac{x^2\Delta^2 u_0}{2!} + \frac{x^3\Delta^3 u_0}{3!} + \ldots\right]$$ (8)

(b) Use Simpson's rule to evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by dividing the interval into eight equal parts. (8)

UNIT – V

5. (a) Use Euler's method to solve $\frac{dy}{dx} = 1 + xy$, given y (0) = 2. Find y (0.1), y(0.2) and y(0.3). (8)

(b) Use Milne's method to find y(0.8) from $\frac{dy}{dx} = 1 + y^2$, given y(0) = 0, y(0.2) = 0.2027, y(0.4) = 0.4228 and y(0.6) = 0.6841. (8)

OR

5. (a) Use Euler's modified method to solve $\frac{dy}{dx} = x^2 + y$, given y(0) = 1 and, determine y(0.02) and y(0.04). (8)

(b) Use Runge-Kutta fourth order method to find the numerical solution at x = 0.8 for $\frac{dy}{dx} = \sqrt{(x+y)}$, given y (0.4) = 0.41. Take step size h = 0.2. (8)