

B.Tech. III Semester (Main/Back) Examination - 2014

Civil Engg.

3CE6(O) Advance Engg. Mathematics

Time : 3 Hours

Maximum Marks : 80

Min. Passing Marks : 24

Instructions to Candidates:

Attempt any **five** questions, selecting **one** question from each **unit**. All questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)

Unit - I

1. a) Find the Fourier series to represent $f(x) = x - x^2$ in the interval $-1 < x < 1$. (8)
- b) Obtain the first three cosine terms and the constant term in the Fourier series for y , where
- | | | | | | | |
|------|---|---|----|---|---|---|
| $x:$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $y:$ | 4 | 8 | 15 | 7 | 6 | 2 |
- (8)

OR

1. a) Use convolution theorem to evaluate $Z^{-1} \left[\frac{z^2}{(z-a)(z-b)} \right]$. (8)
- b) Obtain $Z[\sin n\theta]$, $Z[\cos n\theta]$ and hence find $Z[a^n \sin n\theta]$ and $Z[a^n \cos n\theta]$. (8)

Unit - II

2. a) Find Inverse Laplace transform of:

$$\frac{s}{s^4 + 4a^4} \quad (8)$$

- b) Use L.T. theory to solve:

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = t; \text{ given } y(0) = -3, y(1) = -1. \quad (8)$$

OR

2. a) If $L[f(t)] = \bar{f}(s)$, then prove that $L[tf(t)] = -\frac{d}{ds} \bar{f}(s)$. Hence, find the Laplace transform of: $t^2 \cos at$. (8)

b) Find the bounded solution $y(x,t)$, $0 < x < 1, t > 0$ of the boundary value problem.

$$\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-t}, y(x,0) = x \quad (8)$$

Unit - III

3. a) Find $f(x)$ if its Fourier sine transform is $\frac{1}{s} e^{-as}$. Hence deduce $\bar{F}_s^{-1} \left[\frac{1}{s} \right]$. (8)

b) Use Fourier transform theory to solve:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ given that } u_x(0,t) = 0 \text{ and } u(x,0) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}, u(x,t) \text{ is bounded and } x > 0, t > 0. \quad (8)$$

OR

3. a) Find the Fourier transform of:

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases} \quad (8)$$

b) Use Fourier transform theory to solve:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, t > 0 \text{ given that } u(x,0) = f(x) = \begin{cases} u_0, & |x| < a \\ 0, & |x| > a \end{cases} \quad (8)$$

Unit - IV

4. a) Use Lagrange's formula to find $f(x)$ from the following data.

$x:$	0	1	4	5
$f(x):$	4	3	24	39

(8)

b) A slider in a machine moves along a fixed straight rod. Its distance x (cm.) along the rod is given below for various values of time t (secs.).

$t:$	0	0.1	0.2	0.3	0.4	0.5	0.6
$x:$	30.28	31.43	32.98	33.54	33.97	33.48	32.13

Evaluate:

i) $\frac{dx}{dt}$ for $t=0.1$ and $t=0.3$.

ii) $\frac{dx}{dt}$ for $t=0.5$ and also $\frac{d^2x}{dt^2}$ for all above points. (8)

OR

4. a) Compute $u_{12.2}$ from the following data:

$x:$	10	11	12	13	14
$10^5 u_x:$	23967	28060	31788	35209	38368

State the formula used. Why? (8)

b) Evaluate Numerically $\int_0^1 \frac{dx}{1+x^2}$ using:

i. Simpson's $\frac{1}{3}$ rule and

ii. Simpson's $\frac{3}{8}$ rule. Find approximate value of π . (8)

Unit - V

5. a) Employ Picard's method to obtain correct to four places of decimals solution to the differential equation $\frac{dy}{dx} = x^2 + y^2$ with $y=0$ when $x=0$ for $x=0.4$ (8)

b) Solve by the modified Euler's method the differential equation: $\frac{dy}{dx} = y^2 - \frac{y}{x}$, given that $y=1$ when $x=1$ and determine y for $x=1.1(0.1)1.4$. (8)

OR

5. a) If $\frac{dy}{dx} = x + y^2$, use Runge-Kutta method to find an approximate value of y for $x=0.2$, given that $y=1$ when $x=0$ ($h=0.1$). (8)

b) Use Milne's Predictor-Corrector method to obtain $y(0.4)$ for the following differential equation $\frac{dy}{dx} = 2e^x - y$, given that

$x:$	0	0.1	0.2	0.3
$y:$	2	2.01	2.04	2.09

(8)