1E 2002

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B. Tech. (Semester I) (Main/Back) Examination,

102 (Common to All Branch)

ENGINEERING MATHEMATICS-I

Time : 3 Hours | Min. Passing Marks : 24 | Total Marks : 80

Instruction to Candidates :

Attempt any five questions. Selecting one question from each unit. All questions carry equal marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)

Unit-'I'

1. (a) Find the asymptotes of the following curve: $x^3 - 5x^2y + 8xy^2 - 4y^3 + x^2 + 2y^2 - 3xy - 1 = 0$ [8]

(b) Show that in the parabola $y^2 = 4ax$, the radius of curvature at any point P is $\frac{2(SP)^{3/2}}{\sqrt{a}}$, where S is the focus of the parabola. Also, if ρ_1, ρ_2 are radii of curvature at the extremities of a focal chord of the above parabola, then $(\rho_1)^{-2/3} + (\rho_2)^{-2/3} = (2a)^{-2/3}$ [8]

OR

1. (a) Trace the curve : $y^2(a + x) = x^2(a - x)$ [8]

(b) Trace the curve : $r = a + b\cos\theta,\ a > b$ [8]

Unit-'II'

2. (a) If $u = \sin^{-1}\left(\frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}\right)$, Prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20}\tan u$$ [8]

(b) ABC is an acute-angled triangle with fixed base BC. If δb, δc, δA and δB are small increments in b, c, A and B respectively when the vertex A is given a small displacement δx parallel to BC. Prove that $\delta A = h\delta x\left(\frac{1}{c^2} - \frac{1}{b^2}\right)$, where h is the unaltered height of the triangle. [8]

OR

2. (a) If $u = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$ Show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$ [8]

(b) Find points on the surface $z^2 = xy + 1$ whose distances from the origin are minimum. [8]

Unit-'III'

3. (a) Prove that the surface and volume of the solid generated by the revolution of the tractrix $x = a\cos t + \frac{1}{2}a\log\tan^2\left(\frac{t}{2}\right), y = a\sin t$ about its asymptote are respectively equal to the surface and half the volume of a sphere of radius a. [10]

(b) Find by double integration the area of the region enclosed by the following curves : $y^2 = 4ax$ and $x^2 = 4ay$. [6]

OR

3. (a) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{(x^2+y^2)}}$ by changing the order of integration. [8]

(b) (i) Show that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}\,\sqrt{\frac{1}{n}}}{n\sqrt{\left(\frac{1}{n}+\frac{1}{2}\right)}}$ [4]

(ii) Show that $B(m,n) = \int_0^x \frac{y^{m-1}}{(1+y)^{m+n}}dy$ [4]

Unit-'IV'

4. Solve the following differential equations:

(a) $(1 + y^2)dx = (\tan^{-1} y - x)dy$ [5]

(b) $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$ [6]

(c) $(x - y)^2\frac{dy}{dx} = a^2$ [5]

OR

4. Solve the following differential equations:

(a) $(D^4 + 2D^3 - 3D^2)y = 3e^{2x} + 4\sin x$ [5]

(b) $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin 2x$ [5]

(c) $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x$ [6]

Unit-'V'

5. (a) Solve:

$$x^3\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} + xy = \sin(\log x)$$ [8]

(b) Solve:

$$(2x^2 + 3x)\frac{d^2y}{dx^2} + (6x + 3)\frac{dy}{dx} + 2y = (x + 1)e^x$$ [8]

OR

5. (a) Solve:

$$\cos x\frac{d^2y}{dx^2} + \sin x\frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x$$ [8]

(b) Solve by the method of variation of parameters:

$$(x + 2)\frac{d^2y}{dx^2} - (2x + 5)\frac{dy}{dx} + 2y = (x + 1)e^x$$ [8]