2014

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Explain Frobenius method for solving a second-order ordinary differential equation. 3

(b) Solve the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - m^2)y = 0$$

by Frobenius method. 7

OR

(a) Discuss the method of separation of variables for solving a partial differential equation. 4

(b) Use the method of separation of variables to solve the equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

with boundary conditions

$$y(0,\ t) = 0 = y(l,\ t)$$ 6

2. *(a)* The generating function for Legendre polynomials $P_n(x)$ is

$$g(x,\ t) = \frac{1}{\sqrt{1-2xt+t^2}}$$

such that $\dfrac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} t^n P_n(x)$.

Show that $P_n(-x) = (-1)^n P_n(x)$. 2

(b) Use the generating function for $P_n(x)$ to establish the following recurrence relations : 4+4=8

(i) $(2n+1)xP_n(x) - nP_{n-1}(x) = (n+1)P_{n+1}(x)$

(ii) $P'_{n+1}(x) = P_n(x) + 2xP_n(x) - P'_{n-1}(x)$

OR

(a) For Hermite polynomials $H_n(x)$, show that $H'_n(x) = 2nH_{n-1}(x)$. 3

(b) Deduce the integral representation for the Bessel functions $J_n(x)$ as

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\, d\theta$$

What will be the integral representation of $J_0(x)$? 6+1=7

3. *(a)* Deduce the Cauchy-Reimann conditions for the analyticity of a function of complex variable. 4

(b) State and prove Cauchy integral theorem. 4

(c) Write down the Laurent series expansion of a function of complex variable. 2

OR

(a) A complex function $f(z)$ has the form

$$f(z) = \frac{\phi(z)}{\psi(z)}$$

If z_0 is a simple pole for $f(z)$ such that $\psi(z_0) = 0$ but $\psi'(z_0) \neq 0$, then show that the residue of $f(z)$ at z_0 is given by

$$R = \frac{\phi(z_0)}{\psi'(z_0)}$$

2

(b) If $f(z) = \dfrac{z}{z^2 - 1}$ is a complex function, find its singular points and calculate the residues at those points. 3

(c) Apply residue theorem to evaluate the integral $\int_{-\infty}^{\infty} \dfrac{e^{\alpha x}}{e^x + 1} dx \; (|\alpha| < 1)$ using a rectangular contour. 5

4. (a) Discuss the two sets of unit vectors in a curvilinear coordinate system (u_1, u_2, u_3). 4

(b) Deduce the expressions for the divergence and curl of a vector field in spherical polar coordinates. 3+3=6

OR

(a) Explain what you mean by symmetric and skew-symmetric tensors. 3

(b) Show that contraction of a tensor results in a new tensor of rank 2 less than that of the original tensor. 2

(c) If A^{α} and B_{β} are the components of a contravariant and a covarient vectors respectively, then show that their outer product $A^{\alpha} B_{\beta}$ is a mixed tensor of rank 2. 3

(d) Show that the kronecker delta δ^i_j is a mixed tensor of rank 2. 2

5. *(a)* Show that any square matrix can uniquely be expressed as the sum of a symmetric matrix and a skew-symmetric matrix. 4

(b) If A is a non-singular square matrix of order n, then show that

$$|\text{adj } A| = |A|^{n-1}$$

where $|\text{adj } A|$ is the determinant of adj A. 4

(c) Prove that the product of two or more orthogonal matrices of same rank is also an orthogonal matrix. 2

OR

(a) Solve the following simultaneous equations by matrix method : 4

$$2x+3y+4z=9$$
$$2y+3z=8$$
$$x-z=-3$$

(b) Diagonalize the matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$ 4

(c) Show that $\text{Tr}(AB) = \text{Tr}(BA)$, where A and B are two matrices conformable for the products AB and BA, and $\text{Tr}(AB)$ is the trace of the matrix AB. 2

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2014

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—I)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—I

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. If x_0 is an ordinary point of the second-order ordinary differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

then its power series solution has the form

(a) $\sum_{r=0}^{\infty} a_r(x-x_0)^r$ ( )

(b) $\sum_{r=0}^{\infty} a_r(x-x_0)^{k+r}$ ( )

(c) $\sum_{r=0}^{\infty} a_r(x-x_0)^{k-r}$ ( )

(d) $\sum_{r=0}^{\infty} a_r(x-x_0)^{-k-r}$ ( )

2. Laplace's equation for the scalar V in two-dimensional Cartesian coordinates is

(a) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ ( )

(b) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \rho(x, y)$ ( )

(c) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial V}{\partial y} = 0$ ( )

(d) $\frac{\partial V}{\partial x} + \frac{\partial^2 V}{\partial y^2} = \rho(x, y)$ ( )

3. The expression for $P_2(\cos\theta)$ is

(a) $\cos\theta$ ( )

(b) $\frac{1}{2}\cos^2\theta$ ( )

(c) $\frac{1}{2}(3\cos^2\theta - 1)$ ( )

(d) $\frac{1}{2}(3\cos^2\theta + 1)$ ( )

4. In Hermite polynomial $H_n(x)$, the coefficient of x^n is

(a) n ( )

(b) $2n$ ( )

(c) n^2 ( )

(d) 2^n ( )

5. If C is the circle $|z| = 2$, then the value of the integral $\oint_C \frac{z^2}{z-1} dz$ is

(a) $-2\pi i$ ( )

(b) $2\pi i$ ( )

(c) -2π ( )

(d) 2π ( )

6. If the point z_0 is a pole of order n for a complex function $f(z)$, then the residue of $f(z)$ at z_0 is

(a) $\frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} [(z - z_0)^n f(z)]_{z=z_0}$ ( )

(b) $\frac{1}{n!} \frac{d^n}{dz^n} [(z - z_0)^n f(z)]_{z=z_0}$ ( )

(c) $\frac{1}{(n+1)!} \frac{d^{n+1}}{dz^{n+1}} [(z - z_0)^n f(z)]_{z=z_0}$ ( )

(d) $\frac{1}{(n+1)!} \frac{d^n}{dz^n} [(z - z_0)^n f(z)]_{z=z_0}$ ( )

7. In curvilinear coordinate system (u_1, u_2, u_3), the gradient of a scalar field V is

(a) $\sum_{i=1}^{3} \frac{\hat{e}_i}{h_i} \frac{\partial V}{\partial u_i}$ ( )

(b) $\sum_{i=1}^{3} \hat{e}_i \frac{\partial V}{\partial u_i}$ ( )

(c) $\sum_{i=1}^{3} \hat{e}_i h_i \frac{\partial V}{\partial u_i}$ ( )

(d) $\sum_{i=1}^{3} \hat{e}_i \frac{\partial}{\partial u_i}(h_i V)$ ( )

8. The contour C encloses the singular points $z_1, z_2, z_3, \ldots, z_n$ for the complex function $f(z)$. The residues of $f(z)$ at these points are $R_1, R_2, R_3, \ldots, R_n$ respectively. The value of the integral $\oint_C f(z)\,dz$ is

(a) $\pi i(R_1 + R_2 + R_3 + \ldots + R_n)$ ( )

(b) $2\pi i(R_1 + R_2 + R_3 + \ldots + R_n)$ ( )

(c) $\frac{\pi i}{2}(R_1 + R_2 + R_3 + \ldots + R_n)$ ( )

(d) $\frac{\pi}{2i}(R_1 + R_2 + R_3 + \ldots + R_n)$ ( )

9. The diagonal elements of a skew-symmetric matrix are

(a) all non-zero ( )

(b) all zero ( )

(c) all purely imaginary ( )

(d) all complex ( )

10. The eigenvalues of a square matrix of order 2 are λ_1 and λ_2. The trace of the matrix is

(a) 0 ( )

(b) $\lambda_1 - \lambda_2$ ( )

(c) $\lambda_1 + \lambda_2$ ( )

(d) $\lambda_1 \lambda_2$ ( )

SECTION—II

(*Marks* : 15)

Give short answers to the following questions : 3×5=15

1. Explain what you mean by ordinary points and singular points for a second-order ordinary differential equation.

2. For integral values of n, show that $J_{-n}(x) = (-1)^n J_n(x)$, where $J_n(x)$ is the Bessel function.

3. Identify the singular points for the complex function $f(z) = \dfrac{2z^2}{4 - z^2}$ and calculate the residues of $f(z)$ at those singular points.

4. Explain Einstein's summation convention. Give an example.

5. Show that the eigenvalues of a Hermitian matrix are all real.

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