2014

(5th Semester)

MATHEMATICS

Paper : MATH–351

(Computer-oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Find the negative root of $x^3 - 2x + 5 = 0$ correct to three places of decimals by Newton-Raphson method. 5

(b) Find the function whose first difference is $x^3 + 3x^2 + 5x + 12$. Write the result in factorial polynomial expansion form. 3+2=5

2. *(a)* Find the positive real root of $x \log_{10} x = 1{\cdot}2$ using the bisection method in four iterations. 5

(b) Find the second difference of the polynomial $f(x)=7x^4+12x^3-6x^2+5x-3$ taking $h=2$ giving the result in normal polynomial form. 4+1=5

UNIT—II

3. *(a)* Obtain Newton's divided difference interpolation formula for interpolation with non-equal intervals of the argument. 6

(b) Find $\log_{10}\pi$ by Newton's formula for forward interpolation given that

$$\log_{10} 3{\cdot}141=0{\cdot}4970679364$$
$$\log_{10} 3{\cdot}142=0{\cdot}4972061807$$
$$\log_{10} 3{\cdot}143=0{\cdot}4973443810$$
$$\log_{10} 3{\cdot}144=0{\cdot}4974825374$$
$$\log_{10} 3{\cdot}145=0{\cdot}4976206498$$

using $\pi = 3{\cdot}1415926536$. 4

4. *(a)* Find the equation of the cubic curve which passes through the points (4, – 43), (7, 83), (9, 327), (12, 1053). 4

(b) Obtain Newton's backward interpolation formula for interpolation with equal intervals of the argument. 6

UNIT—III

5. *(a)* Solve the following system of equations by Gaussian elimination method : 6

$$2\cdot 51x+1\cdot 48y+4\cdot 53z=0\cdot 05$$
$$1\cdot 48x+0\cdot 93y-1\cdot 30z=1\cdot 03$$
$$2\cdot 68x+3\cdot 04y-1\cdot 48z=-0\cdot 53$$

(b) Solve the following by Gauss-Jordan method : 4

$$x+y+z=9$$
$$2x-3y+4z=13$$
$$3x+4y+5z=40$$

6. *(a)* Solve $2x+3y=3$; $2x+3y=5$ by Gauss-Seidel iteration method. 4

(b) By Crout's method, solve the following system of simultaneous equations : 6

$$2x-6y+8z=24$$
$$5x+4y-3z=2$$
$$3x+y+2z=16$$

UNIT—IV

7. *(a)* The function $f(x)$ is tabulated in the table below :

x	0·7	0·8	0·9	1·0	1·1
y	0·644218	0·717356	0·783327	0·841471	0·891207

Find $f'(0\cdot 77)$. 4

(b) Obtain the formula for Simpson's one-third rule of integration for numerical integration. 6

8. (a) From the table below, find $f''(0 \cdot 15)$: 6

x	0·0	0·05	0·1	0·15	0·20	0·25	0·30	0·35
y	1·275	1·342	1·517	1·821	1·99	2·45	2·85	3·40

(b) Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ by using trapezoidal rule. 4

UNIT—V

9. (a) Given that

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \quad y(1) = 1$$

Evaluate $y(1 \cdot 3)$ by Euler's multiple-step method. 6

(b) Find the approximate solution of the initial value problem $y' = 1 + y^2$, $y(0) = 0$ by Picard's method and compare it with the exact solution. 4

10. Using any predictor-corrector method, find $y(0 \cdot 4)$ for the differential equation

$$\frac{dy}{dx} = 1 + xy, \quad y(0) = 2$$ 10

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2014

(5th Semester)

MATHEMATICS

Paper : MATH-351

(Computer-oriented Numerical Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(Multiple Choice)

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Which of the following statements is true for difference operator, where Δ, ∇, δ, μ stand for forward, backward, central and average difference operators respectively?

(a) $\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i$ ☐

(b) $\nabla^n y_i = \nabla^n y_{i+1} - \nabla^{n-1} y_{i-1}$ ☐

(c) $\delta f(x) = f\left(x - \frac{h}{2}\right) - f\left(x + \frac{h}{2}\right)$ ☐

(d) $\mu f(x) = f\left(x + \frac{h}{2}\right) + f\left(x - \frac{h}{2}\right)$ ☐

2. A reciprocal factorial function denoted by $x^{(-n)}$, where n is a positive integer is a product of the form given by

(a) $(x+h)(x+2h)(x+3h)\ldots(x+nh)$ ☐

(b) $\dfrac{1}{x(x+h)(x+2h)(x+3h)\ldots(x+nh)}$ ☐

(c) $\dfrac{1}{x(x+h)(x+2h)(x+3h)\ldots[x+(n-1)h]}$ ☐

(d) $\dfrac{1}{(x+h)(x+2h)(x+3h)\ldots(x+nh)}$ ☐

3. While constructing a forward difference table if six arguments are given, the forward difference table will contain terms up to

(a) $\Delta^6 y$ ☐

(b) $\Delta^5 y$ ☐

(c) $\Delta^7 y$ ☐

(d) None of the above ☐

4. If $f(x) = \dfrac{1}{x^2}$, then the divided difference of $\delta(a, b)$ is equal to

(a) $\dfrac{-(a+b)}{a^2b^2}$ ☐

(b) $\dfrac{ab+bc+ac}{a^2b^2c^2}$ ☐

(c) $\dfrac{(a-b)}{a^2b^2}$ ☐

(d) $\dfrac{(b-a)}{a^2b^2}$ ☐

5. The method for obtaining the solution of the system of simultaneous equation by Gauss elimination method depends on reducing the coefficient matrix to a/an

(a) diagonal matrix ☐

(b) lower triangular matrix ☐

(c) upper triangular matrix ☐

(d) diagonally dominant matrix ☐

6. The coefficient matrix obtained from the simultaneous equations

$$a_{11}x + a_{12}y + a_{13}z = d_1,\ b_{21}x + b_{22}y + b_{23}z = d_2,\ c_{31}x + c_{32}y + c_{33}z = d_3$$

will be a diagonally dominant matrix if

(a) $|a_{11}| \geq |a_{12}| + |a_{13}|$
$|b_{21}| \geq |b_{22}| + |b_{23}|$
$|c_{31}| \geq |c_{32}| + |c_{33}|$ ☐

(b) $|a_{11}| \geq |a_{12}| + |a_{13}|$
$|b_{22}| \geq |b_{21}| + |b_{23}|$
$|c_{33}| \geq |c_{31}| + |c_{32}|$ ☐

(c) $|a_{11}| \leq |a_{12}| + |a_{13}|$
$|b_{22}| \leq |b_{21}| + |b_{23}|$
$|c_{33}| \leq |c_{31}| + |c_{32}|$ ☐

(d) $|a_{11}| + |a_{12}| + |a_{13}| \geq |d_1|$
$|b_{21}| + |b_{22}| + |b_{23}| \geq |d_2|$
$|c_{31}| + |c_{32}| + |c_{33}| \geq |d_3|$ ☐

7. In the general quadrature formula, trapezoidal rule is obtained by putting

(a) $n = 2$ ☐

(b) $n = 4$ ☐

(c) $n = 2$ and 4 both ☐

(d) $n = 1$ ☐

8. The value of $y'(5)$ from the table

x	0	1	2	3	4
y	1	1	15	40	85

is

(a) 625 ☐

(b) 6·25 ☐

(c) 0·625 ☐

(d) 0·0625 ☐

9. The formula for solving the first-order differential equation $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$ by Taylor's series method is

(a) $y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$ ☐

(b) $y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \ldots$ ☐

(c) $y_1 = y_0 + hf(x_0, y_0)$ ☐

(d) $y_1 = y_0 + \Delta y$ ☐

10. For solving ordinary differential equation numerically, which among the following is applied if successive derivatives can be obtained easily?

(a) Taylor's method ☐

(b) Picard's method ☐

(c) Euler's method ☐

(d) Runge-Kutta method ☐

SECTION—B

(Very Short Answer)

(*Marks* : 15)

Each question carries 3 marks

1. Evaluate

$$\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$$

where Δ is forward difference operator.

2. Obtain the relation between divided difference and simple difference given by

$$\delta(x_n, x_{n-1}, \ldots, x_3, x_2, x_1, x_0) = \frac{\Delta^n y_0}{n! h^n}$$

where δ denotes divided difference and Δ denotes simple difference.

3. What is diagonally dominant system for a simultaneous equation? Are the given equations diagonally dominant?

$$3x + 9y - 2z = 10$$
$$4x + 2y + 13z = 19$$
$$4x - 2y + z = 3$$

4. Use Simpson's 1/3rd rule to find an approximate value of $\log 2$ from the integral $\int_1^2 \frac{dx}{x}$.

5. Using Runge-Kutta method of second-order, find $y(0 \cdot 1)$ for the differential equation

$$\frac{dy}{dx} = xy + y^2, \quad y(0) = 1$$

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