

2015

(5th Semester)

MATHEMATICS

Paper : MATH-351

(Computer-oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

*The figures in the margin indicate full marks
for the questions*

Answer **any one** question from each Unit

UNIT—I

1. (a) Can we find the real root of the equation $x = 2^x$? In the light of this function, can the real root of $x^3 + x^2 - 1 = 0$ in the interval $[0, 1]$ be obtained by the method of successive iteration? 2+3=5

G16/138a

(Turn Over)

(b) Express

$$3x^4 - 4x^3 + 6x^2 + 2x + 1$$

in terms of a factorial polynomial and find its fourth-order difference. $2+3=5$

2. (a) Write an algorithm for Newton-Raphson formula to find the root of algebraic equation. Further find the cube root of 10 using Newton-Raphson formula. $3+2=5$

(b) If

$$y = \frac{1}{x(x+3)(x+6)}$$

then show that

$$\Delta^2 y = \frac{108}{x(x+3)(x+6)(x+9)(x+12)}$$

where Δ is the forward difference operator. 5

UNIT—II

3. (a) From the data given below, find the number of students whose weight is between 60 and 70 : 4

Weight	:	0-40	40-60	60-80	80-100	100-120
No. of Students	:	250	120	100	70	50

(b) Obtain Newton's divided difference interpolation formula for interpolation with non-equal intervals of the argument. 6

4. (a) Use Lagrange's formula to find a polynomial of degree three which takes the values prescribed below and hence find the value of $f(x)$ when $x = 2$: $4+1=5$

x	:	0	1	3	4
$f(x)$:	-12	0	6	12

(b) The area A of a circle of diameter d is given for the following values :

d	:	80	85	90	95	100
A	:	5026	5674	6362	7088	7854

Find the approximate value for the area of a circle of diameter 97. 5

UNIT—III

5. (a) Write an algorithm for Gauss elimination method. Further show the back substitution procedure in the Gaussian elimination method of solving a simultaneous linear equation. $2+3=5$

(b) By Crout's method, solve the following system of simultaneous equations : 5

$$\begin{aligned}x + y + z &= 9 \\2x - 3y + 4z &= 13 \\3x + 4y + 5z &= 40\end{aligned}$$

6. (a) What is diagonally dominant matrix? Make the following system of equations diagonally dominant and write the corresponding system of simultaneous equations : 2+3=5

$$3x + 9y - 2z = 10$$

$$4x + 2y + 13z = 19$$

$$4x - 2y + z = 3$$

- (b) Solve the system of following linear equations by Gauss-Jordan method : 5

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 5z = 7$$

UNIT—IV

7. (a) Compute by Simpson's rule the value of the integral

$$I = \int_0^1 \frac{x^2}{1+x^2} dx$$

by dividing into four equal parts. 5

- (b) Obtain the general quadrature formula for equidistant points to find the approximate integration of any function for which numerical values are known. 5

8. (a) Obtain the first and second derivatives of the function tabulated below, at the point $x = 0.51$: 5

x :	0.4	0.5	0.6	0.7	0.8
y :	1.5836494	1.7974426	2.0442376	2.3275054	2.6510818

- (b) Compute the value of the integral

$$I = \int_0^{\pi/3} \sqrt{\cos\theta} d\theta$$

by trapezoidal rule. 5

UNIT—V

9. (a) Solve

$$\frac{dy}{dx} = 1 - y, y(0) = 0$$

using Euler's method. Find y at $x = 0.1$ and $x = 0.2$. Compare the result with results of the exact solution. 3+2=5

- (b) Using Taylor's method, find $y(0.1)$ correct to 3-decimal places from

$$\frac{dy}{dx} + 2xy = 1, y_0 = 0 \quad 5$$

10. Using any Predictor-Corrector method, find $y(0.4)$ for the differential equation

$$\frac{dy}{dx} = 1 + xy, y(0) = 2 \quad 10$$

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MATHEMATICS

Paper : MATH-351

(Computer-oriented Numerical Analysis)

(PART : A—OBJECTIVE)

(Marks : 25)

Answer **all** questions

SECTION—A

(Marks : 10)

Each question carries 1 mark

Put a Tick mark against the correct answer in the box provided :

1. By definition of backward difference operator, $\nabla f(x)$ equals to

(a) $f(x+h) - f(x)$

(b) $f(x+h) + f(x)$

(c) $f(x) - f(x+h)$

(d) $f(x) - f(x-h)$

2. If

$$f(x) = \frac{1}{(x+1)(x+2)(x+3)}$$

then the value of $\Delta^3 f(x)$ is

(a) $-60x^{(-6)}$

(b) $12x^{(-5)}$

(c) $360x^{(-7)}$

(d) $-12x^{(-5)}$

3. If $f(0) = 5$, $f(1) = 1$, $f(2) = 9$, $f(3) = 25$ and $f(4) = 55$, then the value of $f(5)$ is

(a) 105

(b) 115

(c) 125

(d) None of the above

4. If $u_1 = 1$, $u_3 = 17$, $u_4 = 43$ and $u_5 = 89$, then the value of u_2 is

(a) 10

(b) 15

(c) 6

(d) 5

5. The method for obtaining the solution of the system of simultaneous equations by Gauss-Jordan elimination method depends on reducing the coefficient matrix to a/an

- (a) lower triangular matrix
- (b) upper triangular matrix
- (c) diagonal matrix
- (d) diagonally dominant matrix

6. The coefficient matrix obtained from the simultaneous equations

$$a_{11}x + a_{12}y + a_{13}z = d_1$$

$$b_{21}x + b_{22}y + b_{23}z = d_2$$

$$c_{31}x + c_{32}y + c_{33}z = d_3$$

will be a diagonally dominant matrix, if

- (a) $|a_{11}| \geq |a_{12}| + |a_{13}|$
 $|b_{21}| \geq |b_{22}| + |b_{23}|$
 $|c_{31}| \geq |c_{32}| + |c_{33}|$
- (b) $|a_{11}| \geq |a_{12}| + |a_{13}|$
 $|b_{22}| \geq |b_{21}| + |b_{23}|$
 $|c_{33}| \geq |c_{31}| + |c_{32}|$
- (c) $|a_{11}| \leq |a_{12}| + |a_{13}|$
 $|b_{22}| \leq |b_{21}| + |b_{23}|$
 $|c_{33}| \leq |c_{31}| + |c_{32}|$
- (d) $|a_{11}| + |a_{12}| + |a_{13}| \geq |d_1|$
 $|b_{21}| + |b_{22}| + |b_{23}| \geq |d_2|$
 $|c_{31}| + |c_{32}| + |c_{33}| \geq |d_3|$

7. In the general quadrature formula, Simpson's 1/3rd rule is obtained by putting

- (a) $n = 1$
- (b) $n = 4$
- (c) $n = 2$
- (d) $n = 2$ and 4 both

8. When numerical integration is applied for the integration of a function of single variable, the method is called

- (a) mechanical quadrature
- (b) general quadrature
- (c) Simpson's $\frac{1}{3}$ rd rule
- (d) trapezoidal rule

9. Taylor series method is a powerful single-step method if we are able to find easily the successive

- (a) integration
- (b) derivatives
- (c) continuity
- (d) partial derivatives

10. Which of the following statements is wrong?
- (a) Modified Euler's method is Runge-Kutta method of second order.
 - (b) Euler's method is the Rung-Kutta method of first order.
 - (c) For solving ordinary differential equation numerically, the most reliable and most accurate method is Runge-Kutta method.
 - (d) For solving ordinary differential equation numerically, Euler's method needs h to be very large to get a reasonable accuracy.

SECTION—B

(Marks : 15)

Each question carries 3 marks

1. Obtain the relation $\nabla^n f(x + nh) = \Delta^n f(x)$, where Δ is the forward difference operator, and ∇ is the backward difference operator.

2. Show that the divided differences are independent of the order of arguments, i.e., $\delta(x_0, x_1) = \delta(x_1, x_0)$. Is it true for more than two arguments also?

3. Solve the given equations by Gauss elimination method :

$$x + y = 2; 2x + 3y = 5$$

4. Find the value of $\log 2^{1/3}$ from

$$\int_0^1 \frac{x^2}{1+x^3} dx$$

using Simpson's $\frac{1}{3}$ rd rule with $h = 0.25$.

5. Using Picard's method, solve the differential equation

$$\frac{dy}{dx} = 1 + xy \text{ with } y(0) = 2$$

and find $y(0.2)$.

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- (a) mechanical quadrature
- (b) general quadrature
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- (d) trapezoidal rule

9. Taylor series method is a powerful single-step method if we are able to find easily the successive

- (a) integrals
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- (c) continuities
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