

2015

(5th Semester)

MATHEMATICS

Paper : MATH-353

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

*The figures in the margin indicate full marks
for the questions*

Answer **one** question from each Unit

UNIT—I

1. (a) Show that the modulus of sum of two complex numbers is always less than or equal to the sum of their moduli. 5
- (b) Find the centre and radius of the circle passing through the points $1, i, 1+i$. 5

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(Turn Over)

2. (a) If z_1 and z_2 are two complex numbers, then prove that

$$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2$$

if and only if $z_1 \bar{z}_2$ is purely imaginary. 5

- (b) If z_1, z_2, z_3 are the vertices of an isoceles triangle, right angled at the vertex z_2 , then prove that

$$z_1^2 + z_2^2 + z_3^2 = 2(z_1 + z_3)z_2 \quad 5$$

UNIT-II

3. (a) For what value of z the function defined by the equation

$$z = \sin u \cosh v + i \cos u \sinh v, \omega = u + iv$$

ceases to be analytic? 5

- (b) Show that the function $f(z) = xy + iy$ is everywhere continuous but not analytic. 5

4. (a) If $u = x^3 - 3xy^2$, show that there exists a function $v(x, y)$ such that $\omega = u + iv$ is an analytic function in a finite region. 5

- (b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at that point. 5

(3)

UNIT—III

5. (a) State and prove Cauchy-Hadamard formula for the radius of convergence. 4

(b) Find the radius of convergence of the following power series : 6

$$(i) \sum \frac{z^n}{2^n + 1}$$

$$(ii) 1 + \frac{a \cdot b}{1 \cdot c} z + \frac{a \cdot (a+1) b \cdot (b+1)}{1 \cdot 2 \cdot c \cdot (c+1)} z^2 + \dots$$

6. (a) Find the domain of convergence of the power series

$$\sum \left(\frac{2i}{z+1+i} \right)^n \quad 5$$

(b) Find the region of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3 4^n} \quad 5$$

UNIT—IV

7. (a) Using the definition of the integral of $f(z)$ on a given path, evaluate

$$\int_{-2+i}^{5+3i} z^3 dz \quad 5$$

(b) If $f(z)$ is analytic within and on a closed contour C and a is any point within C , then show that

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z) dz}{(z-a)} \quad 5$$

8. (a) Write down the Cauchy's integral formula for the derivative of an analytic function; hence show that for a function $f(z)$ which is analytic in a region D and if $f(z)$ has, at any point $z=a$ of D , derivatives of all orders, all of which are again analytic functions in D , their values are given by

$$f^n(a) = \frac{1}{2\pi i} \int_C \frac{f(z) dz}{(z-a)^{n+1}}$$

where C is any closed contour in D surrounding the point $z=a$.

6

- (b) Evaluate by Cauchy integral formula

$$\int_C \frac{z dz}{(9-z^2)(z+i)} \quad 4$$

UNIT—V

9. (a) Expand $\frac{z+3}{z(z^2-z-2)}$ for the region $|z| > 2$. 4

- (b) Examine the nature of the following functions : 6

(i) $\frac{1}{1-e^z}$ at $z=2\pi i$

(ii) $\frac{1}{\sin z - \cos z}$ at $z=\frac{\pi}{4}$

10. State and prove Liouville's theorem. Use this result to prove the fundamental theorem of algebra. 1+4+5=10

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V/MAT (vii)

2015

(5th Semester)

MATHEMATICS

Paper : MATH-353

(Complex Analysis)

(PART : A—OBJECTIVE)

(Marks : 25)

The figures in the margin indicate full marks for the questions

SECTION--A

(Marks : 10)

Each question carries 1 mark

Put a Tick mark against the correct answer in the box provided :

1. If \bar{z} is the conjugate of z , then

- (a) $|z| > |\bar{z}|$
- (b) $|z| < |\bar{z}|$
- (c) $|z| = |\bar{z}|$
- (d) $|z| = -|\bar{z}|$

2. The real part of $\frac{5}{-3+4i}$ is

(a) $\frac{3}{5}$

(b) $-\frac{3}{5}$

(c) $-\frac{4}{5}$

(d) $\frac{4}{5}$

3. The analytic function whose imaginary part is $e^x \cos y$ is

(a) e^z

(b) ie^z

(c) ie^{-z}

(d) e^{-z}

4. The function $\sin x \cosh y + i \cos x \sinh y$ is

(a) neither continuous nor analytic

(b) continuous but not analytic

(c) continuous as well as analytic everywhere

(d) not analytic everywhere

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4. The function $\sin x \cosh y + i \cos x \sinh y$ is

(a) neither continuous nor analytic

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(c) continuous as well as analytic everywhere

(d) not analytic everywhere

5. If $\lim_{n \rightarrow \infty} |u_r|^{1/n} = l$, then the series $\sum u_n$ is absolutely convergent for

- (a) $l > 1$
- (b) $l < 1$
- (c) $l = 1$
- (d) $l \geq 1$

6. The power series $\sum n z^n$ will converge

- (a) if $z = 0$
- (b) if $|z| = 1$
- (c) if $|z| > 1$
- (d) for all real values of z

7. A Jordan curve consisting of continuous chain of a finite number of regular arcs is called a

- (a) continuous arc
- (b) contour
- (c) rectifiable arc
- (d) multiple arc

8. The value of the integral $\int_C \frac{dz}{z-a}$, while C is the circle $|z-a|=r$ is

- (a) 2π $|z| < 1$ (a)
- (b) -2π $|z| > 1$ (d)
- (c) $2\pi i$ $|z| = 1$ (b)
- (d) $-2\pi i$ $|z| \leq 1$ (c)

9. The function $\frac{z+1}{z(z-2)}$ has/have singularity/singularities at

- (a) $z=0$ only $0 < |z| < 1$ (a)
- (b) $z=2$ only $|z| = 2$ (d)
- (c) $z=0$ and $z=2$ only $1 < |z| < 2$ (c)
- (d) $z=-1$ only s to consider last line (b)

10. The nature of the function $\frac{\sin(z-a)}{(z-a)}$ at $z=a$ is

- (a) removable singularity removable (a)
- (b) non-isolated singularity non-isolated (d)
- (c) isolated singularity isolated (c)
- (d) pole pole (b)

(5)

SECTION—B
(Marks : 15)

(a) *Each question carries 3 marks*

• Answer **all** questions

Answer the following :

1. Prove that for any complex number z , $|z|^2 = z\bar{z}$, where \bar{z} is the conjugate of z .

6. The polar form $Z = r(\cos \theta + i \sin \theta)$ will satisfy which of the following?

(a) $|Z| = 1$

(c) $|Z| = 1$

(d) for all real values of θ

7. A Jordan curve consisting of continuous chain of a finite number of regular arcs is called a

(a) continuous arc

(b) contour

(c) rectifiable arc

(d) multiple arc

- 2.** With suitable example, show that continuity is not a sufficient condition for the existence of a finite derivative.

3. Examine the convergence of the series $\sum z^n$.

(Full Question)

MATHEMATICS

Paper : MATH-353

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(PART-I—OBJECTIVE)

(Marks : 25)

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SECTION-A

(Maximum 10)

Each question carries 1 mark.

Put a Tick mark against the correct answer in the box provided :

1. If \bar{z} is the conjugate of z , then

(a) $|z| > |\bar{z}|$

(b) $|z| < |\bar{z}|$

(c) $|z| = |\bar{z}|$

(d) $|z| = -|\bar{z}|$

4. Show that $\int_C \frac{dz}{z} = 2\pi i$, where C is a complete circle.

3. The analytic function whose imaginary part is $e^x \cos y$ is

- (a) $e^x + \square$
- (b) $e^x - \square$
- (c) $e^x - \square$
- (d) $e^x - \square$

4. The function $\sin x \cosh y + i \cos x \sinh y$ is

- (a) neither continuous nor analytic
- (b) continuous but not analytic
- (c) continuous as well as analytic everywhere
- (d) not analytic everywhere

5. Define non-isolated singularity with a suitable example.

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