GANPAT UNIVERSITY
M.Sc. Third Semester Examination (C.B.C.S) Nov-Dec, 2014
Subject: Mathematics
Paper: MMA 301 FAS Functional Analysis

Time: 3 hours Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1 (a) Let E be an orthonormal subset of inner product space X. Show that for each $x \in X$, the set $E_x = \{u \in E : <x, u> \neq 0\}$ is a countable set. [7]

(b) Prove that an infinite dimensional Hilbert space is separable iff it has a countable orthonormal basis. [7]

Q-2 (a) State and prove Riesz Representation theorem. Can we drop the completeness in this theorem ? Justify it. [7]

(b) Let G be a subspace of a Hilbert space H and $g \in G'$. Then there is a unique element $f \in H'$ such that $f(x) = g(x); \forall x \in G$ and $\|f\| = \|g\|$. [7]

Q-3 (a) Prove that every bounded sequence in a Hilbert space has a weakly convergent subsequence. [7]

(b) Let E be a non empty convex subset of inner product space X and $x \in X$. If y_1, y_2 are two best approximation from E to x then $y_1 = y_2$. [7]

Q-4 (a) Define Orthonormal set and orthogonal complement. Prove that an Orthonormal set is linearly independent. [3]

(b) State and prove Bessel's inequality. [4]

SECTION: II

Q-5 (a) Let H be a Hilbert space and $T \in BL(H)$. Define the adjoint T^* of T. Show that it is exist. [7]

(b) Let H be a Hilbert space and $T \in BL(H)$. Prove that T is unitary iff T is onto isometry [7]

Q-6 (a) Let H be a Hilbert space and $T \in BL(H)$. Prove that T is bounded below iff $R(T^*)$ is dense in H. [7]

(b) Define Compact Operator on a Hilbert space H and let T be a compact operator. Then show that T is bounded and for each $S \in BL(H)$, ST & TS are compact. [7]

Q-7 (a) Let E be a non empty closed convex subset of Hilbert space X and $x \in X$. Then there is a unique element $y_0 \in E$ which is the best approximation from E to x. [7]

(b) Prove that every Hilbert-schmidt operator is compact and also its self adjoint is a Hilbert-schmidt operator. [7]

Q-8 (a) Prove that limit of sequence of self-adjoint, normal; unitary operator is also self-adjoint, normal, unitary operator respectively. [3]

(b) Prove that product of unitary operator is unitary. Is its sum? Justify [4]

--------END OF PAPER--------

GANPAT UNIVERSITY
M.Sc. Third Semester Examination (C.B.C.S) Nov-Dec, 2014
Subject: Mathematical Modeling
Paper: MMA 302 MMG

Time: 3 hours **Total Marks: 70**

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1	(A) Derive a simple business model to determine an equilibrium price. When the equilibrium price does not exist ? How would you determine a maximum profit ?	07
	(B) Derive "The Break Even Point model". Under which condition the Break Even point does not exist ?	07
Q-2	Discuss the Prey-Predator model in details.	14
Q-3	(A) Derive S.I. model in details. (7 marks)	07
	(B) State Newton's Law of cooling and derive a model which explain " At which rate the temperature of body decreses".	07
Q-4	Write answer of following questions.	
	1. Derive a radioactive Decay model and determine the half life time.	04
	2. Discuss the classification and limitation of Mathematical modeling.	03

SECTION: II

Q-5	(A) Derive the fundamental diagramme of a road traffic. Also find capacity of a road.	07
	(B) Derive S.I.R. model.	07
Q-6	(A) Develop a model to derive an expression for the distance travelled by a car between the moment the driver seens an emergency and the moment the car comes a complete stop.	07
	(B) State and prove Hardy-Weinberg law for genetics. Also derive the H.W. ratio.	07
Q-7	(A) Discuss the microscopic model.	07
	(B) Explain the Lanchester's combat model. Why it is called Hyperbolic square model.	07
Q-8	Write answer of following questions.	
	1.Derive relationship among three fundamental traffic variables. Also derive the continuity equation by principle of conservations of vehicles.	04
	2. If the population consists of only recessive then after a fifth and tenth generation what will be the probability of dominant and Hybrid.	03

--------END OF PAPER--------

GANPAT UNIVERSITY
M.Sc. Third Semester Examination (C.B.C.S) Nov-Dec, 2014
Subject: Mathematics
Paper: MMA 303 NTH Number Theory

Time: 3 hours | Total Marks: 70

Instructions:
1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each section in separate answer book.

SECTION: I

Q-1
1. State and prove fundamental theorem of arithmetic. 07
2. Describe the general form of Euclidean algorithm for finding greatest common divisor of given two integers a, b ; $(b > 0)$. 07

Q-2
1. The congruence relation $ax \equiv b (\mathrm{mod}\, m)$, $(a,m)=1$ has only one solution. 07
2. State and prove Chinese remainder theorem. 07

Q-3
1. State and prove Möbius inversion formula. 07
2. Solve the congruence equation by using Diophantine equation $49x \equiv 47 (\mathrm{mod}\, 81)$. 07

Q-4
Write answer of following questions.
1. If $F_n = 2^{2^n} + 1$, $n > 1$ is a prime then 2 is not a primitive root of F_n. 03
2. A man has 987 oranges. In how many ways can he distribute these among 13 women and 5 children? If all women and children receive equal number of oranges. 04

SECTION: II

Q-5

1. State and prove Wilson's theorem. Does converse hold? 07
2. State and prove Lagrange's theorem. 07

Q-6

1. The Diophantine equation $x^4 + y^4 = z^2$ has no solution in positive integers x, y, z. 07
2. Let m be a negative square-free rational integer. The field $\mathbb{Q}(\sqrt{m})$ has units ± 1, and these are the only units except in the cases $m = -1$ and $m = -3$. The units for $\mathbb{Q}(i)$ are ± 1 and $\pm i$. The units for $\mathbb{Q}(\sqrt{-3})$ are ± 1, $(1 \pm \sqrt{-3})/2$ and $(-1 \pm \sqrt{-3})/2$. 07

Q-7

1. State and prove Fermat's theorem. 07
2. Solve $4x^9 \equiv 7(\text{mod}\,13)$. 07

Q-8

Write answer of following questions.

1. Find the minimal polynomial of the following algebraic numbers 03

   (a) $(1 + \sqrt[3]{7})/2$ (b) $(1 + \sqrt{2} + \sqrt{3})$.

2. Find the remainder whenever 2^{340} is divided by 341. 04

--------END OF PAPER--------

No of printed pages: 2

GANPAT UNIVERSITY

M.Sc. Third Semester Examination (C. B. C. S) Nov-Dec, 2014

Subject: Mathematics

Paper: MMA 304 DGY Differential Geometry

Time: 3 hours | Maximum Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each question in separate answer book.

SECTION: I

Q.1

(a) Let $\overline{\gamma}, \tilde{\gamma} : (a, b) \to \mathbb{R}^3$ be unit-speed curves with same curvature and same torsion. Show that there is a rigid motion M of $\mathbb{R}^3$ such that $\tilde{\gamma}(s) = M(\overline{\gamma}(s))$ for all $s \in (a, b)$. [7]

(b) Define *reparametrization* of a parametrized curve $\overline{\gamma} : (a, b) \to \mathbb{R}^n$. Show that a parametrized curve is regular curve if and only if it has a unit-speed reparametrization. [7]

Q.2

(a) Let $\overline{\gamma}$ be a simple closed curve, let ℓ be its length, and let $\mathcal{A}$ be the area of interior of $\overline{\gamma}$. Then show that $\ell^2 \geq 4\pi\mathcal{A}$, with equality holding iff $\overline{\gamma}$ is a circle. [7]

(b) Define *surface*. Show that $\{(x, y, z) : x^2 + y^2 = 1\}$ is a smooth surface. [7]

Q.3

(a) Show that the quadric $x^2 + y^2 - 2z^2 - \frac{2}{3}xy + 4z = 3$ is a hyperboloid with one sheet. is a hyperboloid of one sheet [7]

(b) Define *curvature* and *torsion* of a unit speed curve. Compute the curvature and torsion of $\overline{\gamma}(s) = \left(a\cos(\frac{s}{\sqrt{a^2+b^2}}), a\sin(\frac{s}{\sqrt{a^2+b^2}}), \frac{bs}{\sqrt{a^2+b^2}}\right)$, $s \in \mathbb{R}$, where $a, b > 0$. [7]

Q.4 Write answers of the following questions.

1. State the *Inverse Function Theorem*. [3]
2. Compute the arc-length of the logarithmic spiral $\overline{\gamma}(t) = (e^{kt}\cos t, e^{kt}\sin t)$ starting at the point $(1, 0)$. [4]

SECTION: II

Q.5

(a) Define *first fundamental form* of a surface $\mathcal{S}$ at a point p on it. Compute the first fundamental form of $\sigma(u, v) = (\sinh u \sinh v, \sinh u \cosh v, \sinh u)$ [7]

(b) Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be smooth surfaces. Show that a smooth map $f : \mathcal{S}_1 \to \mathcal{S}_2$ is a local isometry if and only if the symmetric bilinear maps $\langle \cdot, \cdot \rangle_p$ and $f^*\langle \cdot, \cdot \rangle_p$ on $T_p\mathcal{S}_1$ are equal for every $p \in \mathcal{S}_1$. [7]

Q.6

(a) Define *second fundamental form* of a surface. Compute the second fundamental form of the surface $\sigma(u,v) = (u, v, u^2 + v^2)$ [7]

(b) Compute the principal curvatures and the principal vectors of the surface $\sigma(u,v) = (u, v, v^2 - u^2)$ at $(0,0,0)$. [7]

Q.7

(a) Define *geodesic*. Determine the geodesics of the cylinder $\sigma(u,v) = (\cos u, \sin u, v)$. [7]

(b) State the *Gauss Bonnet theorem*. Show that the sum of interior angles of a regular n-gon on a plane is $(n-2)\pi$. [7]

Q.8 Write answers of the following questions.

1. Define *Christiffel symbols* of of second kind for a smooth surface. [3]
2. Define *Gauss map* and the *Weingarten map*. [4]

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