GANPAT UNIVERSITY

M.Sc. Third Semester Examination (C.B.C.S) Nov-Dec, 2013

Subject: Mathematics

Paper: MMA 301 FAS Functional Analysis

Time: 3 hours

Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1(a) Define an inner product space and prove Bessel's inequality. [07]

(b) Prove that every non zero Hilbert space has an orthonormal basis. Give an example to show that the completeness is necessary in Riesz-Fischer theorem. [07]

Q-2. State and prove Projection theorem. Also show that the completeness of the space cannot be dropped in the theorem. [14]

Q-3(a) State and prove unique Hahn-Banach extension theorem. [07]

(b) Let E be a non-empty closed convex subset of a Hilbert space H. Then prove that for each $x \in H$ there is a unique best approximation from E to x. [07]

Q-4. Write answer of following questions.

1.State and prove Polarization identity for an inner product space. [03]

2. Define weakly convergence in an inner product space. Show that convergence in norms implies weak convergence. Is it converse true? [04]

SECTION: II

Q-5(a) Let H be a Hilbert space and $T \in BL(H)$. Show that there is a unique operator $T^* \in BL(H)$ such that $< T(x), y > = < x, T^*(y) >; \forall x, y \in H$. What can you say if the space is not complete? [07]

(b) Let H be a Hilbert space and $T \in BL(H)$. Prove that T is bounded below iff $R(T^*)$ is dense in H. [07]

Q-6(a) Let H be a Hilbert space and $T \in BL(H)$. If T is self adjoint, then $\|T\| = sup.\{ \mid < T(x), x > \mid : x \in H, \|x\| \leq 1\}$. [07]

(b) Prove that limit of a sequence of compact operators on Hilbert space is compact. [07]

Q-7(a) Prove that every Hilbert-schmidt operator is compact and also its self adjoint is a Hilbert-schmidt operator. [07]

(b) Let H be a Hilbert space and $T \in BL(H)$. Prove that T is compact iff T^* is compact. [07]

Q-8. Write answer of following questions.

1.Define best approximation .Let E be subset of an inner product space X and $x \in \bar{E}$. Then there is a best approximation from E to x iff $x \in E$. [03]

2.Define bounded operator, self adjoint operator, normal operator, unitary operator, compact operator, Hilbert-schmidt operator. [04]

--------END OF PAPER--------

GANPAT UNIVERSITY

M.Sc. Third Semester Examination (C.B.C.S) Nov-Dec, 2013

Subject: Mathematics

Paper: MMA 302 MMG Mathematical Modeling

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1(a) Derive a Simple Business Model to determine an equilibrium price. When the equilibrium price does not exist? How would you determine a maximum profit? Explain with suitable example. 07

(b) Derive a model for Seasonal Agriculture products. 07

Q-2.(a) Derive the Allen's Speculative Model. 07

(b) Derive Samuelsson's Investment Model and show that capital oscillates abuts an equilibrium value. 07

Q-3(a) Derive the logistic of population growth model. Why is so called logistic? 07

(b) Explain the impact of the medicine on the blood in the human body. 07

Q-4. Write answer of following questions.

1.The demand is given by the function $D = 200(15 - 2p)$. Total cost is given by the function $T_c = 3000 + 2D$. Then find the price at the maximum profit. 03

2.Show that even a single infective person can make every susceptible people becomes infective. 04

SECTION: II

Q-5(a)	Discuss the Epidemic Model S.I. with removal class.	07
(b)	State and prove Hardy Weinberg law for Genetics also derive the Hardy-Weinberg ratio.	07
Q-6(a)	Explain the lanchester's combat model with winning case. Why is so called Hyperbolic square model.	07
(b)	Derive 'The fundamental diagram of road traffic.' Also find the capacity of a road.	07
Q-7(a)	Derive the compartment model for Diabetes Mellitus.	07
(b)	Derive a model to find stopping distance.	07
Q-8.	Write answer of following questions.	
	1.Suppose Mahesh has blood group A and his wife Meena has blood group B. If their first child has blood group B, then what are the possibility for the blood group of subsequences child.	03
	2.Write the possible blood groups of the father in terms of blood group of mother and child blood groups.	04

--------END OF PAPER--------

GANPAT UNIVERSITY

M.Sc. [redacted]/Third Semester Examination (C.B.C.S) Nov-Dec, 2013

Subject: Mathematics

MMA 303 NTH NUMBER THEORY

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each section in separate answer book.
3. Figures to the right indicate marks of the respective question.

SECTION : I

Q.1 (i) Find $d = (1106, 497)$ and $x, y \in \mathbf{Z}$ such that $d = 1106x + 497y$. [5]

(ii) Prove that if $n > 2$, then $\phi(n)$ is even and $\phi(n) = \phi(2n)$ if n is odd. [5]

(iii) Find the remainder when 72^{1001} is divided by 31. [4]

Q.2 (i) Using indices, find all the solutions of $7x^{23} \equiv 9 (\bmod 13)$. [6]

(ii) Prove that there exist infinitely many primes. [5]

(iii) Prove that $\tau(n)$ is odd if and only if n is a perfect square. [3]

Q.3 (i) State and prove Euler's Theorem. [6]

(ii) Prove that the last digit of every even perfect number is 6 or 8. [5]

(iii) Prove that product of three positive consecutive integers is divisible by $3!$. [3]

Q.4 (i) Find the smallest positive integer that leaves remainder 3 when divided by 6, 4 when divided by 11 and 5 when divided by 17. [4]

(ii) Prove that the Mobius function $\mu(n)$ is multiplicative. [3]

SECTION : II

Q.5 (i) Find all the primitive roots of 26 . [7]

(ii) State and prove Lagrange's Theorem . [7]

Q.6 (i) Find the general solution of the Pell's equation $x^2 - 47y^2 = 1$. [7]

(ii) Classify all the units of quadratic fields $Q(\sqrt{m}), m < 0$. [7]

Q.7 (i) Prove that the fields $Q(\sqrt{m})$, *for* m = -1, -2, -3, -7, 2, 3 are Euclidean . [6]

(ii) Using continued fractions, solve the Diophantine equation 739x +139y = 3 . [5]

(iii) Verify whether $1 + \sqrt{2} + \sqrt{3}\, i$ is an algebraic integer or not . [3]

Q.8 (i) If a, b are integers of opposite parity with a > b > 0 and (a,b) = 1, prove that ($2ab, a^2 - b^2, a^2 + b^2$) is a primitive solution of $x^2 + y^2 = z^2$. [4]

(ii) Prove that 3 is a Gaussian prime but not a prime in $Q(\sqrt{6})$. [3]

END OF PAPER

GANPAT UNIVERSITY
M.Sc. Third Semester Examination (C.B.C.S) Nov-Dec, 2013
Subject: Mathematics
Paper: MMA 304 DGY Differential Geometry

Time: 3 hours Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1(a) State and prove Fundamental curve theory. [07]

(b) Prove that a parametric curve has a unit speed reparameterization iff it is regular. [07]

Q-2(a) State and prove Isoperimetric inequality. When does the equality hold? [07]

(b) Calculate the first order magnitude and first fundamental form of plane and define the terms Defeomorphism, local defeomorphism, isometric and local isometric. [07]

Q-3(a) Let $f: S_1 \to S_2$ be a local defeomorphism then f is conformal iff there is a function $\lambda: S_1 \to \mathbb{R}$ such that $f^* <\cdot,\cdot>_p = \lambda(p) <\cdot,\cdot>_p$. ($\forall p \in S_1$) [07]

(b) Define surface area and compute it for torus. [07]

Q-4. Write answer of following questions.

1. Find all vertices of the curve $\bar{r} = (a \cos t, b \sin t)$. [03]

2. Derive Serret-Frenet Formulae. [04]

SECTION: II

Q-5(a) State and prove Meusnier's theorem. [07]

(b) Define Gaussian and Mean curvature. Determine the matrices of w_p with respect to a basis $\{\sigma_u, \sigma_v\}$ of T_pS . [07]

Q-6(a) State and prove Euler's theorem for normal curvature. [07]

(b) Define Christoffel symbols and determine at least two in terms of first order magnitudes. [07]

Q-7(a) Find the Gaussian and Mean curvature for surface of revolution. [07]

(b) Derive Codazzi Mainaradi equations. [07]

Q-8. Write answer of following questions.

1. Prove that the map $f^* <\cdot,\cdot>_p$ is symmetric bilinear. [03]

2. Define second fundamental form, second order magnitude, normal and principal curvature. [04]

--------END OF PAPER--------

GANPAT UNIVERSITY
M.Sc. Third Semester Examination (C.B.C.S) Nov-Dec, 2013
Subject: Mathematics
Paper: MMB 305 MME Mathematical Methods

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1(a) Discuss about Interior Dirichlet problem for a circle. 07

(b) State and Prove Parseval's Identity. 07

Q-2(a) Obtain the Fourier Integral Representation of 07

$$f(x) = \begin{cases} 1, & -1 \le x \le 1 \\ 0, & otherwise \end{cases}$$

And Hence deduce that $\int_0^\infty \frac{sins}{s} ds = \frac{\pi}{2}$.

(b) Evaluate $\int_0^\infty \frac{sinx}{x(x^2+a^2)} dx$. 07

Q-3(a) Find $F\left[e^{-ax^2-bx}\right]$, where a>0, b$\in R$. 07

(b) Solve by method of Laplace Transforms, the equation $y'' + 9y = cos2t$, subject to constrain $y(0) = y'(0) = 1$. 07

Q-4 Write answer of following questions.

1.Find L $[sinat\ sinhbt]$ 03

2.State Fubini Toneli Theorem and Define Fourier Integral Representation. 04

SECTION: II

Q-5(a)	Find the Laplace Transform of Periodic function.	07
(b)	Solve: $u_t = 3u_{xx},\ x > 0,\ t > 0$ $subject\ to\ u\left(\frac{\pi}{2}, t\right) = 0,\ \ u_x(0,t) = 0\ and\ u(x,o) = 30cos5x$	07
Q-6(a)	Show that Green's function has the symmetric property.	07
(b)	Show that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$ and deduce $H_2(x)$.	07
Q-7(a)	Derive the Green's function for the Diffusion Equation.	07
(b)	State Gram-Schmidt's Orthonormalization theorem, Define: Orthogonal set, Inner product space , orthonormal set.	07
Q-8	Write answer of following questions. 1. Find L$[te^{-t}(sint)^2]$.	03
	2. Show that Hermit Polynomial satisfies Hermit Differential equation.	04

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