GANPAT UNIVERSITY

M.Sc. Second Semester Examination (C.B.C.S) April-May, 2014

Subject: Mathematics

Paper: MMA 201 RAS Real Analysis

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

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## SECTION: I

Q-1.(a) Prove that outer measure of a closed interval is its length. [07]

(b) Prove that any sequence in an algebra can be considered to be disjoint. [07]

Q-2.(a) Show that $E$ is measurable iff there is a $G_\delta$ – set $G$ with $E \subset G$ and $m^*(G - E) = 0$. [07]

(b) Prove that sum and product of two measurable functions are also measurable. [07]

Q-3.(a) State and prove Littlewood's third principle. [07]

(b) Under what situation a bounded function on [a,b] is Riemann integrable? Justify. [07]

Q-4.(a) State and prove Fatou's lemma. [03]

(b) State Bounded convergence theorem and hence evaluate $\lim_{n\to\infty} \int_2^5 \frac{nx}{1+n^2x^2}\, dx$ . [04]

## SECTION: II

Q-5.(a) Show that every absolutely continuous function is the indefinite integral of its derivative. [07]

(b) Prove that the Lebesgue integral of a non-negative measurable function generates a countably additive measure. [07]

Q-6.(a) State and Prove LDCT. Explain its meaning. [07]

(b) Suppose $f$ is bounded measurable on $[a, b]$ and $F(x) = \int_a^x f(t)dt + F(a)$. Then show that $F' = f\ a.e.\ on\ [a, b]$. [07]

Q-7.(a) Suppose $f$ is absolutely continuous function and $f'(x) = 0\ a.e.\ on\ [a, b]$. Then show that $f$ reduces to a constant. [07]

(b) Define convergence in measure.Give an example of it. Show that if $f_n \to f$ in measure then there is a subsequence $\{f_{n_k}\}$ of $\{f_n\}$ converging to $f\ a.e.$ [07]

Q-8.(a) Define Function of bounded variation and show that monotonically increasing functions are of bounded variation. [03]

(b) State and prove Jordan Lemma. [04]

--------END OF PAPER--------

GANPAT UNIVERSITY

**M.Sc. Mathematics Sem-II Examination April-May, 2014**

**Course: MMA 202 GTP General Topology**

Time: 3 hours ] [Total Marks: 70

Instructions:

1. Standard notations and usual conventions are followed.
2. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
3. Answer each section in separate answer book.

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**SECTION: I**

**Q.1** **(a)** Define lower limit Topology. Show that the lower limit topology on R is strictly finer than the standard topology on R. **[07]**

**(b)** Show that product of two Hausdorff spaces if Hausdorff **[07]**

**Q.2** **(a)** Let X and Y be metric spaces and f a mapping of X into Y. Then show that f is continuous if and only if $x_n \to x \Leftrightarrow f(x_n) \to f(x)$. **[07]**

**(b)** Let $X_n$ be a convergent sequence in a topological space $X$ .Show that it converges to a unique limit if $X$ is Hausdorff. **[07]**

**Q.3** **(a)** With usual notation show that $\overline{A} = A \cup A'$ . **[07]**

(b) State and prove the pasting lemma. Give an example of it. **[07]**

**Q.4** **Write answer of following questions.**

(a) Define Int(A); interior of a set And Bd(A); boundary of a set A. **[03]**

(b) Show that (i) $Int(A) \cap Bd(A) = \Phi$ (ii) $Int(A) \cup Bd(A) = A$ . **[04]**

## SECTION:II

**Q.5** (a) If $X$ and $Y$ are connected spaces then prove that the product space $X \times Y$ is connected. Deduce that the finite product is also connected. **[07]**

(b) let $A$ be a connected subspace of $X$ .If $A \subset B \subset \bar{A}$ ,then show that $B$ is also connected. **[07]**

**Q.6** (a). Define a path connected space. Show that a path connected space is connected. Give an example to show that the converse is not true in general. **[07]**

(b) If $X$ and $Y$ are compact spaces then prove that the product space $X \times Y$ is compact. **[07]**

**Q.7** (a) Define components and path components of a topological space $X$.Show that path components of $X$ are path connected disjoint subspaces of $X$ whose union is $X$ such that each nonempty path connected subspace of $X$ intersect only one of them. **[07]**

(b) If the space $X$is not compact and $Y$ is its one point compactification then prove that $\bar{X} = Y$. **[07]**

**Q.8** **Answer the following questions.**

(a) Let (X,d) be a metric space. Show that $d_1$ defined by $d_1(x,y)= d(x,y)/ [1 + d(x,y)]$ is also a metric on X. Is $(X,d_1)$ a bounded metric? Justify your answer? **[04]**

(b) Show that the union of two connected sets is connected if they have a common point. **[03]**

--------**END OF PAPER**--------

GANPAT UNIVERSITY

M.Sc. Second Semester Examination (C.B.C.S) April-May, 2014

Subject: Mathematics

Paper: MMA 203 FSV Functions of Several Variables

Time: 3 hours

Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

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## SECTION: I

Q-1. Prove that $|<x,y>| \le |x||y|; \forall x,y \in R^n$ and show that equality holds if and only if $x$ and $y$ are linearly dependent. 14

Q-2.(a) If $f: R^n \to R^m$ is differentiable at $a \in R^n$, then there is a unique linear transformation $\lambda: R^n \to R^m$ such that $Df(a) = \lambda$. 07

(b) Prove that $f: R^n \to R^m$ is differentiable at $a \in R^n$ if and only if each $f^i$ is differentiable at $a \in R^n$ $(1 \le i \le m)$ and $Df(a) = (Df^1(a), Df^2(a), \ldots\ldots, Df^m(a))$ 07

Q-3.(a) If $f: R^2 \to R$ defined by $f(x,y) = \sin(xy^2), ((x,y) \in R^2)$ .Then find $f'(a,b)$ by using Chain rule. 07

(b) Let $f: A \to R$ be a bounded function and $a \in R$. Then prove that $f$ is continuous at $a$ if and only if $O(f,a) = 0$ 07

Q-4 . Write answer of following questions.

1. Let $A \subseteq R^n$ be a rectangle and let $f: A \to R^n$ be continuously differentiable. If there is a number $M > 0$ such that $|D_j f^i(x)| \le M$ $(\forall x \in A^o)$ then prove that $|f(x) - f(y)| \le n^2 M |x - y|$ $(\forall x, y \in A)$ 03

2. Let $A \subseteq R^n$ be an open set and $a \in A$. If the maximum (or minimum) of $f: A \to R$ occurs at $a \in A$ and $D_i f(a)$ exists then prove that $D_i f(a) = 0$. Is it converse true? 04

## SECTION: II

Q-5.(a) Define: k-tensor, Tensor product, alternating k-tensor, Alt of k-tensor, Wedge product, Vector field, k-forms. 07

(b) Prove that (i) $f^*(S+T)=f^*S+f^*T$ 07

(ii) $f^*(S\otimes T)=f^*S\otimes f^*T$

Q-6.(a) If $\omega\in\Lambda^k(V)$ then prove that $Alt(\omega)=\omega$. 07

(b) If $\omega\in\Lambda^k(V)$, $\eta\in\Lambda^l(V)$, $\mu\in\Lambda^m(V)$ then prove that 07

$(\omega\wedge\eta)\wedge\mu=\omega\wedge(\eta\wedge\mu)$

Q-7.(a) Prove in usual notation $\dim\left(\Lambda^k(V)\right)=\binom{n}{k}$. 07

(b) Show that $df$ is a 1-form on $R^n$. Let $f:R^n\to R^m$ be a differentiable function. Let $F=f_*^*$ then 07

$$F(dx^i)=\sum_{j=1}^{n}D_j f^i dx^j \quad (1\le i\le m).$$

Q-8. Write answer of following questions.

1. If $f:R^n\to R^m$ is differentiable at $a\in R^n$, then each $D_j f^i(a)$ exists for $1\le i\le m, 1\le j\le n$ and $f'(a)=\left[D_j f^i(a)\right]_{m\times n}$. 03

2. Let $f:R^2\to R^3$ defined by $f(x)=(x^1,x^1+x^2,(x^2)^2)$ where $x=(x^1,x^2)\in R^2$. Find $f'(b)$ where $b=(1,0)\in R^2$ 04

--------END OF PAPER--------

Seat No. :........

GANPAT UNIVERSITY

M.Sc. Second Semester Examination (C.B.C.S) April-May, 2014

Subject: Mathematics

Paper: MMA204MPD (Method of Partial Differential Equations)

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each section in separate answer book.

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## SECTION: I

Q-1 [A] Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^y$. **07**

[B] Solve $(x^2D^2 - y^2D'^2 + xD - yD')z = \log x$. **07**

Q-2 [A] Classify the equation and convert it into canonical form $4r - y^6 t = 3y^5 q$. **07**

[B] Reduce $x^2 \frac{\partial^2 z}{\partial x^2} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and Solve. **07**

Q-3 [A] Solve $r - t\cos^2 x + p\tan x = 0$. By Monge's Method. **07**

[B] Solve $x^2 r + 2xys + y^2 t = 0$. By Monge's Method. **07**

Q-4 Write answer of following questions.

[A] Eliminate the arbitrary functions $f$ and $g$ from $z = f(x - y) + g(x + y)$. **03**

[B] Prove that if $\alpha D + \beta D' + \gamma$ is a factor of $F(D, D')$ $(\alpha \neq 0)$, then $e^{-\frac{\gamma}{\alpha}x} \varphi(\beta x - \alpha y)$ is a solution of $F(D, D')z = 0$, where $\varphi$ is an arbitrary function of a single variable $\xi$. **04**

## SECTION: II

| | | | |
|---|---|---|---|
| Q-5 | [A] | Solve $3r + 4s + t + (rt - s^2) = 1$. By Monge's Method. | 07 |
| | [B] | Discuss the Neumann interior B.V.P. for a circle. | 07 |
| Q-6 | [A] | Use Separation of variable to solve equation $\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t}$, Given that V = 0, when t→ ∞ as well as v = 0 at $x = 0$ and $x = l$. | 07 |
| | [B] | State Harnack's Theorem and Prove it. | 07 |
| Q-7 | [A] | State and prove maximum principle. | 07 |
| | [B] | Transformation of Laplace equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$ into Cylindrical Coordinates. | 07 |
| Q-8 | | Write answer of following questions. | |
| | [A] | State Green's theorem. | 03 |
| | [B] | Solve $r + t - (rt - s^2) = 1$. By Monge's Method. | 04 |

--------END OF PAPER--------

GANPAT UNIVERSITY

M.Sc. **Second** Semester Examination (C.B.C.S) April-May, 2014

Subject: Applied Linear Algebra

Paper: MMB-205 ALA

Time: 3 hours

Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

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## SECTION: I

Q-1 (A) Prove that if $T:U\to V$ is a homomorphism then 07

(i) $KerT$ Is a subspace of $U$

(ii) $T$ Is an isomorphism iff $KerT=\{0_v\}$

(B) If $W_1$ and $W_2$ are subspaces of a finite dimensional vector space $V$ then show that 07

$\dim(W_1+W_2)=\dim W_1+\dim W_2-\dim(W_1\cap W_2)$

Q-2 (A) Show that if $V$ is a finite dimensional vector space over $F$ then $T\in A(V)$ which is right invertible need not imply left invertible. 07

(B)Prove that If $V$ be a finite dimensional vector space over $F$ and $T\in A(V)$ then $T$ is regular iff $T$ is onto. 07

Q-3 (A) Prove that If $V$ is a finite dimensional vector space over $F$ then any two bases of $V$ have the same number of elements. 07

(B) Show that If $p(x)$ is minimal polynomial for $T\in A(V)$ and $S\in A(V)$ is invertible then $p(x)$ is also minimal polynomial for $STS^{-1}$ 07

Q-4 Write answer of following questions.

1. If $\lambda\in F$ is a characteristic root of $T\in A(V)$, then show that $\lambda$ is a root of the minimal polynomial of $T$. 03
2. Find the matrix $D$ with respect to basis $1,x,x^2+x^3$. 04

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## SECTION: II

Q-5 (A) For $A = (\alpha_{ij})_{n\times n} \in F_n$, $B = (\beta_{ij})_{n\times n} \in F_n$ and $\lambda \in F$, show that 07

1. $tr(\lambda A) = \lambda trA$
2. $tr(A + B) = trA + trB$
3. $tr(AB) = tr(BA)$.

(B) If $\dim_F V = n$ and $T \in A(V)$ has all its characteristic roots in $F$ then show that $T$ satisfies a polynomial of degree $n$ over $F$. 07

Q-6 (A) Let $V$ be a finite dimensional vector space over $F$ and $T \in A(V)$. Suppose $V = V_1 \oplus V_2$, where $V_1, V_2$ are subspaces of $V$ invariant under $T$. Let $T/V_1 \equiv T_1$, $T/V_2 \equiv T_2$ and minimal polynomial for $T_i$ be $p_i(x) \in F[x], i \in \{1,2\}$ then show that minimal polynomial for $T$ over $F$ is least common multiple of $p_1(x), p_2(x)$. 07

(B) Prove that If $T \in A(V)$ is unitary iff $T$ takes an orthonormal basis of $V$ into an orthonormal basis of $V$ 07

Q-7 (A) Show that $T \in A(V)$ Is unitary iff $T.T^* = Id$ 07

(B) Prove that If $T$ is Hermition and $T^k(v) = 0$, for some $k \geq 1$ then $T(v) = 0$ 07

Q-8 Write answer of following questions.

1. Show that $tr(AB)$ need not be equal to $trA.trB$ 03
2. For all $A, B \in F_n$ and $\lambda \in F$, show that 04

1. $(A + B) = A + B$ 2. $(\lambda A)' = \lambda A$.