GANPAT UNIVERSITY

M.Sc. Second Semester Examination (C.B.C.S) April-May, 2013

Subject: Mathematics

Paper: MMA 201 RAS Real Analysis

Time: 3 hours — Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each section in separate answer book.

SECTION: I

Q.1 **(a)** Prove that arbitrary intersection of algebras is an algebra. **[07]**

(b) Prove that Lebesgue measure is countably additive measure. **[07]**

Q.2 **(a)** Show that sum and product of two measurable functions is measurable. **[07]**

(b) State Littlewood's three principles and prove third principle. **[07]**

Q.3 **(a)** State and prove Fatou's lemma. **[07]**

(b) State and prove Lebesgue's monotone convergence theorem and hence evaluate $\lim_{n\to\infty}\int_2^5 \frac{nx}{1+nx}dx$. **[07]**

Q.4 **Write answer of following questions.**

1. Prove that every borel set is measurable set. **[03]**

2. Show χ_E is measurable if and only if E is measurable. **[04]**

SECTION: II

Q.5 State and prove Lebesgue's dominated convergence theorem and hence evaluate $\lim_{n\to\infty}\int_2^5 \frac{nx}{1+n^2x^2}dx$. **[14]**

Q.6 **(a)** Show that f is of bounded variation on $[a,b]$ if and only if f is the difference of two monotonically increasing real valued functions on $[a,b]$. **[07]**

(b) Let $f, g \in BV[a,b]$ then prove that **[07]**

(i) $f \cdot g \in BV[a,b]$.

(ii) $\frac{f}{g} \in BV[a,b]$ if $g \neq 0$.

Q.7 **(a)** Let f be integrable function on $[a,b]$. Then prove that the indefinite integral of f is a continuous function of bounded variation on $[a,b]$. **[07]**

(b) Prove that every absolutely continuous function is the indefinite integral of its derivative. **[07]**

Q.8 **Write answer of following questions.**

1. Show that the absolutely continuous function is function of bounded variation **[03]**

2. Define convergence in measure and establish relation with convergence a.e. **[04]**

--------**END OF PAPER**--------

GANPAT UNIVERSITY

M.Sc. Mathematics Sem-II Examination April-May, 2013

PAPER : **MMA 202 GTP General Topology**

Time: 3 hours] [Total Marks: 70

Instructions:

1. Standard notations and usual conventions are followed.
2. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
3. Answer each section in separate answer book.

SECTION: I

Q.1 **(a)** Define lower limit Topology. Let τ be the collection of subset of X , whose complements is either finite or all of X .Then prove that (X,τ) is a topological space. **[07]**

(b)

(i) Prove or disprove: Arbitrary intersection of open sets is open. **[07]**

(ii) Show that A is open in X if and only if A contains a nhbd of each of its points.

Q.2 **(a)** Show that $f:(X,\tau_1)\to(Y,\tau_2)$ is continuous if and only if $f(\overline{A})\subset\overline{f(A)}$, for every subset A of X . **[07]**

(b) Let $f_1:A\to X, f_2:A\to Y$ and $f:A\to X\times Y$ be functions such that $f(a)=(f_1(a),f_2(a))$ where $A, X, and\, Y$ are topological spaces. **[07]**

Prove that f is continuous if and only if f_1 *and* f_2 are continuous.

Q.3 **(a)** If β is a basis for the topology of X and C is a basis for the topology of Y, show that $\{B\times c \mid B\in\beta\ and\ c\in C\}$ is a basis for the product topology of $X\times Y$. **[07]**

(b) State and prove the pasting lemma. Give an example of it. **[07]**

Q.4 **Write answer of following questions.**

(a) 1.Prove that $(-1,1)$ and R are homeomorphic to each other. **[03]**

2. Prove that $Int.(A)$ is the largest open set contained in A . **[02]**

(b) Prove that product of two Housdorff space is a Housdorff space.

(c) Let (X,d) be a metric space. If $\overline{d}:X\times X\to\mathbb{R}$ is defined as $\overline{d}(x,y)=\min\{d(x,y),1\}$ then show that $\overline{d}$ is a metric . **[02]**

SECTION:II

Q.5 **(a)** Shrow that both Connectedness and Compactness are topologycal property. **[07]**

(b) Define a path connected space. Show that Path Connected space is Connected. Give an example to show that the converse is not true in general. **[07]**

Q.6 **(a).** If A is a connected subset of X and $A \subset B \subset \overline{A}$ then prove that B is connected. Deduce that in particular $\overline{A}$ is also connected. **[07]**

(b) Prove that Connectedness is finite productive . **[07]**

Q.7 **(a)** Prove that product of two compact space is compact. **[07]**

(b) Let X be a locally compact Hansdorff space which is not compact. Define one-point compactification of X . If Y is this compactification , show that Y is a compact Hausdorff space such that X is dense in Y . **[07]**

Q.8 **Answer the following questions.**

1. **Let X be a** Topological space and a,b ∈ X., If $b \in comp(a)$ then prove that $comp(b) = comp(a)$. Is the converse true ?. Justify your answer. **[02]**
2. Let $X = U \bigcup V$ & Y is connected subspace of X then show that either $Y \subset U$ or $Y \subset V$. **[03]**
3. Show that union of two connected set is connected if they have a common point. **[02]**

--------**END OF PAPER**--------

GANPAT UNIVERSITY

M.Sc. Second Semester Examination (C.B.C.S.) April-May, 2013

Subject: Mathematics

Paper: MMA 203 FSV Functions of Several Variables

Time: 3 hours **Total Marks: 70**

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each section in separate answer book.

SECTION - I

Q.1 A Show that a closed and bounded subset of $\mathbb{R}^n$ is bounded. **07**

B Define the oscillation $o(f,a)$ of a bounded function $f: A \to R$ at $a \in A \subset R^n$. If $f: A \to R$ is a bounded function, prove that f is continuous at $a \in A$ if and only if $o(f,a) = 0$. **07**

Q.2 A Define derivative of a mapping $f: \mathbb{R}^n \to \mathbb{R}^m$. Show that every linear transformation is differentiable. **07**

B Define a continuously differentiable mapping $f: \mathbb{R}^n \to \mathbb{R}^m$ at $a \in \mathbb{R}^n$. If $f: \mathbb{R}^n \to \mathbb{R}^m$ is a continuously differentiable mapping at $a \in \mathbb{R}^n$, show that $Df(a)$ exists. **07**

Q.3 A If a mapping $f: \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $a \in \mathbb{R}^n$, show that $D_j f^i(a)$ exists for all $1 \le i \le m$ and for all $1 \le j \le n$. Find $f'(a)$. **07**

B For differentiable mappings $f: \mathbb{R}^n \to \mathbb{R}^m$ and $g: \mathbb{R}^m \to \mathbb{R}^p$, prove the Chain Rule. **07**

Q.4 Write answer of the following questions.

A Discuss the continuity of the mapping $f: \mathbb{R}^n \to \mathbb{R}$ given by $f((0,0)) = 0$, and $f((x,y)) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right)$ for all $(x,y) \ne (0,0)$. **03**

B Find the derivative of $f(x,y,z) = x + \sin y \sin z$ at the point $a = (0,0,0)$. **04**

SECTION - II

Q.5 A Define the directional derivative of a function $f:\mathbb{R}^n \to \mathbb{R}$ at $a \in \mathbb{R}^n$. **07**
In usual notations, prove that $D_{x+y}f(a) = D_x f(a) + D_y f(a)$.

B Find all the partial derivatives of $f(x,y,z) = \left(x^{y+z}, (x+y)^z\right)$. **07**
Determine the Jacobian matrix $f'(a)$ of f at the point $a = (1,1,1)$.

Q.6 A Define the term k-tensor on a vector space V. In usual notations, prove that $\mathfrak{I}^k(V)$ of all k-tensors is a vector space. **07**

B Define the alternating tensor. For $w \in \Lambda^k(V)$, show that $Alt(w) = w$. **07**

Q.7 A Define a vector field F on $\mathbb{R}^n$. If F and G are vector fields F on $\mathbb{R}^n$, then define $F+G$ and show that it is also a vector field. **07**

B Define a k-form on $\mathbb{R}^n$. Let $f:\mathbb{R}^n \to \mathbb{R}$ be a differentiable. **07**
Define df and show that it is a 1-form on $\mathbb{R}^n$.

Q.8 Write answer of the following questions.

A For $x = (x^1, x^2)$, $y = (y^1, y^2) \in \mathbb{R}^2$, define $T(x,y) = x^1y^1 - x^2y^1$; find $Alt(T)$. **03**

B For $x = (x^1, x^2) \in \mathbb{R}^2$, define $\varphi(x) = x^1 + x^2$ and $\eta(x) = 2x^1$; find $\varphi \wedge \eta$. **04**

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GANPAT UNIVERSITY
M.Sc. Second Semester Examination (C.B.C.S) April-May, 2013
Subject: Mathematics
Paper: MMA204MPD (Method of Partial Differential Equations)

Time: 3 hours
Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1 [A] Using the method of Undetermined Coefficients, Solve the Differential Equation is $y'' + 4y = 8x^2$. 07

[B] Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2\sin(\log x)$ Using by Cauchy-Euler linear Differential Equation. 07

Q-2 [A] Solve $xr + ys = 10xy^3 - p$. By Lagrange Method. 07

[B] Reduce the canonical form and Solve 07

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0.$$

Q-3 [A] Solve $r = a^2 t$. By Monge's Method. 07

[B] Solve $r + 3s + t + (rt - s^2) = 1$. By Monge's Method. 07

Q-4 Write answer of following questions.

[A] Solve the method of Variation Parameter 03

$$\frac{d^2y}{dx^2} + a^2 y = secax.$$

[B] Reduce $\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$ to canonical form. 04

SECTION: II

Q-5 [A] Solve $y^2r - 2ys + t = p + 6y$. By Monge's Method 07

[B] Derive General Solution of one - dimensional Wave equation $\frac{\partial^2 y}{\partial x^2} = \left(\frac{1}{c^2}\right)\left(\frac{\partial^2 y}{\partial t^2}\right)$. 07

Q-6 [A] Use Separation of variable to solve equation $\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t}$, Given that V = 0, when $t \to \infty$ as well as v = 0 at $x = 0$ and $x = l$. 07

[B] Transformation of Laplace equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ into Polar Coordinates. 07

Q-7 [A] Derive Dirichlet Problem in Rectangle. 07

[B] State Harnack's Theorem and Prove it. 07

Q-8 Write answer of following questions.

[A] Solve $2s + (rt - s^2) = 1$. By Monge's Method. 03

[B] Using Separation of variable to solve $\frac{\partial u}{\partial x} = 2\left(\frac{\partial u}{\partial t}\right) + u$, Where $u(x, 0) = 6e^{-3x}$. 04

--------END OF PAPER--------

GANPAT UNIVERSITY
M.Sc. Second Semester Examination (C.B.C.S) April-May, 2013
Subject: Mathematics
Paper: MMB 205 ALA Applied Linear Algebra

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q.1 **(a)** Let $T:U\to V$ be linear homomorphism and $W=\ker(T)$. If T is onto then U/W is isomorphic to V. **[07]**

(b) Let V be a finite dimensional vector space over F. If W_1 and W_2 are subspaces of V then prove that $(W_1^\circ)^\circ = W_1$ and $(W_1+W_2)^\circ = W_1^\circ \cap W_2^\circ$. **[07]**

Q.2 **(a)** Let V be a finite dimensional vector space over F. Then prove that $T\in A(V)$ is invertible if and only if T is onto. **[07]**

(b) Let V be a finite dimensional vector space over F, then prove that $r(ST)\le \min\{r(S),r(T)\}$, for $S,T\in A(V)$. **[07]**

Q.3 **(a)** Let V be a finite dimensional vector space over F, then prove that $T\in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero. **[07]**

(b) Let W_1 and W_2 be two subspace of a finite dimentional vector space V. Then prove that $\dim(W_1+W_2)=\dim(W_1)+\dim(W_2)-\dim(W_1\cap W_2)$. **[07]**

Q.4 **Write answer of following questions.**

1. Show that span $\{(1,0,1),(-1,2,3),(0,1,-1)\}$ is all of R^3. **[03]**

2. Let W_1 and W_2 are subspace of a vector space V. Then show that $W_1\cap W_2$ and W_1+W_2 are subspace of V. **[04]**

SECTION: II

Q.5 **(a)** If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$. **[07]**

(b) Show that $\det(AB) = \det(A) \cdot \det(B)$, for $A, B \in F_n$. Is it true for $A, B \in F_n$ $\det(A+B) = \det(A) + \det(B)$? **[07]**

Q.6 **(a)** Prove that the determinant of a triangular matrix is the product of its entries on the main diagonal. **[07]**

(b) For each $i = 1, 2, \ldots\ldots, k$ $V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus \ldots\ldots \oplus V_k$. Then prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$. **[07]**

Q.7 **(a)** If two rows of any matrix are equal then prove that its determinant is zero. **[07]**

(b) If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular. **[07]**

Q.8 **Write answer of following questions.**

1. Define trace of a matrix. Let $A, B \in M_n(F)$ then show that $tr(AB) = tr(BA)$. **[03]**

2. State Cramer's rule and hence solve the following system. **[04]**

$$x + 2y + 3z = -5$$
$$2x + y + z = -7$$
$$x + y + z = 0$$

--------**END OF PAPER**--------