# GANPAT UNIVERSITY

## M.Sc. First Semester Examination (C.B.C.S) Nov-Dec, 2014

## Subject: Mathematics

## Paper: MMA 101 CAN Complex Analysis

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

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### SECTION: I

**Q-1** (a)Let $f$ be analytic with $f'(z) = 0$ at every point of a domain D. Then prove that $f$ reduces to a constant. [7]

(b)Determine an analytic function having real part as $y + e^x \cos y$. [7]

**Q-2** (a)Let $f$ be continuous at $z_0$ and $f(z_0) \neq 0$. Then there exist a neighborhood of $z_0$ on which $f$ will be different from zero. Also Evaluate $(-8i)^{\frac{1}{5}}$. [7]

(b)State and prove Cauchy Goursat's theorem. [7]

**Q-3** (a)Show that Cauchy Riemann equations are necessary for the analyticity of a function but they are not sufficient. Also derive the complex form of Cauchy Riemann equations. [7]

(b)Suppose $f$ is continuous on a domain D and the integral is independent of the path. Then show that $f$ has an antiderivative on D. [7]

**Q-4** (a)Find out the locus of z for which $|z - 4i| + |z + 4i| = 10$. [3]

(b)Prove that every analytic function determines an orthogonal system of curves. Also prove that $e^x[(x^2 - y^2)\cos y - 2xy \sin y]$ is harmonic. [4]

### SECTION: II

**Q-5** (a) Explain types of singularity with an example. [7]

(b) Evaluate $\int_0^{\infty} \frac{2x^2 - 1}{x^4 + 5x^2 + 4} dx$ [7]

**Q-6** (a)State and prove the fundamental theorem of algebra. [7]

(b)Prove that non constant entire function must be unbounded. [7]

**Q-7** (a)State and prove principle of deformation of paths [7]

(b)State and prove Gauss Mean Value theorem. [7]

**Q-8** (a) State Cauchy's integral formula and hence evaluate $\int \frac{e^{2z}}{(z-1)(z-2)} dz$ over the contour $|z|=3$. [3]

(b) Find out the Laurent's series expansion of $\frac{-1}{(z-1)(z-2)}$, where $1 < |z| < 2$ [4]

--------END OF PAPER--------

GANPAT UNIVERSITY
M.Sc. First Semester Examination (C.B.C.S) Nov-Dec, 2014
Subject: Mathematics
Paper: MMA 102 ALB Algebra

Time: 3 hours | Total Marks: 70

Instructions:
1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each section in separate answer book.

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## SECTION: I

| | | |
|---|---|---|
| Q-1 | 1. A subgroup $N$ of group $G$ is normal in $G$ if and only if $Na = aN$, for $a \in G$. | 07 |
| | 2. State and prove fundamental theorem of homomorphism. | 07 |
| Q-2 | 1. State and prove Cayley's theorem. | 07 |
| | 2. If $G$ is a finite group then $c_a = \dfrac{O(G)}{O(N_a)}$. | 07 |
| Q-3 | 1. State and prove Lagrange's theorem. | 07 |
| | 2. Every finite abelian group is the direct product of cyclic groups. | 07 |
| Q-4 | Write answer of following questions. | |
| | 1. The group $G$ is solvable if and only if $G^k = \{e\}$, for some integer $k$. | 03 |
| | 2. Prove that group of order 72 is not a simple group. | 04 |

## SECTION: II

**Q-5**

1. A finite integral domain is a field. 07
2. Let $R$ be a commutative ring with unity and $A$ be an ideal of $R$ then $R/A$ is a field if and only if $A$ is a maximal ideal. 07

**Q-6**

1. Let $F$ be a field and $p(x) \in F[x]$ then $< p(x) >$ is a maximal ideal in $F[x]$ if and only if $p(x)$ is irreducible over $F$. 07
2. Every principal ideal domain is a unique factorization domain. 07

**Q-7**

Every integral domain can be imbedded in a field. 14

**Q-8**

Write answer of following questions.

1. State and prove Gauss's lemma. 03
2. Show that there exists an irreducible in $Z[\sqrt{-3}]$ which is not a prime. 04

--------END OF PAPER--------

## GANPAT UNIVERSITY
## M.Sc. First Semester Examination (C.B.C.S) Nov-Dec, 2014
## Subject: Mathematics
## Paper: MMA 103 MDE Methods of Differential Equations

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

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### SECTION: I

Q-1 (a) Define regular singular point. Find Frobenius series expansion $2x^2y''+x(2x+1)y'-y=0$ near regular singular point. [7]

(b) If $y_1(x)\,\&\,y_2(x)$ are two linearly dependent solution of $y''+P(x)y'+Q(x)y=0$ on [a,b] iff $W(y_1(x),y_2(x))=0$. [7]

Q-2 (a) State and prove orthogonality of Legendre polynomials. [7]

(b) Prove that (i) $(n+1)P_{n+1}=(2n+1)xP_n-nP_{n-1}$ (ii) $nP_n=xP_n'-P_{n-1}'$. [7]

Q-3 (a) Discuss the behavior of singular solution of Bessel's equation and find its indicial equation. [7]

(b) State only Rodrigue's formula and find Legendre polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$. Also express $x^3+5x+2$ in terms of Legendre polynomials. [7]

Q-4 (a) Show that all the roots of $P_n(x)$ are real and lies between -1 and 1. [3]

(b) Prove that $\frac{d}{dx}\left[x^nJ_n(x)\right]=x^nJ_{n-1}(x)$ [4]

### SECTION: II

Q-5 (a) State and prove orthogonality of Bessel's function. [7]

(b) Solve by Picard's method (i) $y'=y; y(0)=1$ (ii) $y'=x+y; y(0)=1$ [7]

Q-6 (a) Prove that (i) $(1+x)^p=F(-p,b,b,x)$ (ii) $e^x=\lim\limits_{b\to\infty}F\left(a,b,a,\frac{x}{b}\right)$ [7]

(iii) $\cos x=\lim\limits_{a\to\infty}F\left(a,a,\frac{1}{2},\frac{-x^2}{4a^2}\right)$

(b) State and prove necessary and sufficient condition for compatibility of partial differential equation $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ [7]

Q-7 (a) (i)Solve Quasi-linear equation $\frac{y^2z}{x}p+xzq=y^2$ [7]

(ii) Solve : $yzp+zxq=xy$ by Langrange's method.

(b) Solve: (i) $2(x^2+x)y''+(1+5x)y'+y=0; x=0$ (ii) $(x^2-x-6)y''+(5+3x)y'+y=0; x=3$ [7]

Q-8 (a) Find complete integral of $q=3p^2$. [3]

(b) Solve: $z=px+qy+p^2+q^2$ by Charpit's method. [4]

--------END OF PAPER--------

GANPAT UNIVERSITY

M.Sc. **First** Semester Examination (C.B.C.S) Nov-Dec, 2014

Subject: Mathematics

**Paper: 104: Operations Research**

Time: 3 hours Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer books.

**SECTION: I**

Q-1 a. State and prove fundamental theorem of linear programming. 07

b. Solve the following L.P.P. by Simplex Method : 07

Max $Z = 2x_1 + 3x_2 + 10x_3$

subject to $x_1 + 2x_3 = 0, x_2 + x_3 = 1$ and $x_1, x_2, x_3 \geq 0$.

Q-2 a. Describe North-West corner method for a transportation problem. 07

b. Solve the following assignment problem using Hungarian method. 07

| | A | B | C | D |
|---|---|---|---|---|
| I | 1 | 4 | 6 | 3 |
| II | 9 | 7 | 10 | 9 |
| III | 4 | 5 | 11 | 7 |
| IV | 8 | 7 | 8 | 5 |

Q-3 a. Obtain an initial BFS to the following TP using North-West Corner method. Also find the cost involved in the solution you obtained. 07

| | D1 | D2 | D3 | D4 | Availability |
|---|---|---|---|---|---|
| O1 | 11 | 13 | 17 | 14 | 250 |
| O2 | 16 | 18 | 14 | 10 | 300 |
| O3 | 21 | 24 | 13 | 10 | 400 |
| Demand | 200 | 225 | 275 | 250 | |

b. Solve the game whose payoff matrix is given by 07

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{ccc} B1 & B2 & B3 \end{array} \\ \text{Player A} \begin{array}{c} A1 \\ A2 \\ A3 \end{array} & \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & 1 \end{bmatrix} \end{array}$$

Q-4 Write answer of following questions.

a. Describe the meaning of payoff matrix. 03

b. Explain the term "dummy source" in context of a transportation problem. 04

## SECTION: II

Q-5 a. What is meant by a saddle point? Discuss the method for determining saddle point; and also discuss the rules for game without saddle point to reduce the size of payoff matrix. 07

b. Solve the following game using matrix method 07

PLAYER B

| PLAYER A | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 1 | 7 | 2 |
| $A_2$ | 6 | 2 | 7 |
| $A_3$ | 5 | 1 | 6 |

Q-6 a. Using the graphical Method, calculate the total elapsed time needed to complete both jobs on five machines. 07

MACHINES

| Job | | | | | | |
|---|---|---|---|---|---|---|
| Job 1 | Sequence | A | B | C | D | E |
| | Time(hours) | 3 | 4 | 2 | 6 | 2 |
| Job 2 | Sequence | B | C | A | D | E |
| | Time(hours) | 5 | 4 | 3 | 2 | 6 |

b. Define: Event, Network, Critical Path, Backward Pass, Slack of an activity. 07

Q-7 a. Explain the method for processing n jobs through m machines. 07

b. There are five jobs, each of which must go through machines A, B, C in the order $A \rightarrow B \rightarrow C$. Processing times are given in the following table 07

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| Job i | Processing Time | | |
|---|---|---|---|
| | $A_i$ | $B_i$ | $C_i$ |
| 1 | 8 | 5 | 4 |
| 2 | 10 | 6 | 9 |
| 3 | 6 | 2 | 8 |
| 4 | 7 | 3 | 6 |
| 5 | 11 | 4 | 5 |

Find a sequence that minimized the total elapsed time.

Q-8 Write answer of following questions.

1.Describe the meaning of concurrent activities 03

2. Draw a network diagram of activities for the project. 04

| ACTIVITY | PREDESSOR ACTIVITY |
|---|---|
| A | ---- |
| B | A |
| C | A |
| D | B |
| E | A |
| F | B, E |
| G | C |
| H | D,F |
| I | G |
| J | H, I |

--------END OF PAPER--------

GANPAT UNIVERSITY
M.Sc. First Semester Examination (C.B.C.S) Nov-Dec, 2014
Subject: Mathematics
Paper: MMB 105 GTH Graph Theory

Time: 3 hours Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory.
2. Answer each section in separate answer book.

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## SECTION: I

| | | |
|---|---|---|
| Q-1 | 1. A connected graph $G$ is an Euler graph if and only if it can be decomposed into circuits. | 07 |
| | 2. A graph is a tree if and only if it is minimally connected. | 07 |
| Q-2 | 1. Every connected graph has at least one spanning tree. | 07 |
| | 2. Every circuit has an even number of edges in common with any cut-set. | 07 |
| Q-3 | 1. A tree with $n$-vertices has $(n-1)$ edges. | 07 |
| | 2. An arborescence is a tree in which every vertex other than the root has in-degree of exactly one. | 07 |
| Q-4 | Write answer of following questions. | |
| | 1. State seating arrangement problem. Also give the solution with their respective diagram. | 03 |
| | 3. Define with examples: | 04 |
| | (a) Hamiltonian graph (b) Vector degree | |
| | (c) Condensation (d) Arborescence | |

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## SECTION: II

Q-5

1. Every tree with two or more vertices is $2-\textit{chromatic}$. 07
2. The vertices of every planar graph can be properly colored with five colors. 07

Q-6

1. Let $G$ be a graph. A matching $M$ in $G$ is a maximum matching if and only if $G$ has no $M$ - augmenting path. 07
2. If $G$ is a bipartite graph then the maximum size of a matching in $G$ equals the minimum size of a vertex cover of $G$. 07

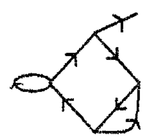
Q-7

1. A graph of $n$-vertices is a complete graph if and only if it's chromatic polynomial can be given by $P_n(\lambda)=\lambda.(\lambda-1).(\lambda-2)....(\lambda-n+1)$ . 07
2. State and prove Hall's matching condition. 07

Q-8

Write answer of following questions.

1. Find the adjacency matrix of the below digraph: 03



2. Define with examples: 04

(a) Chromatic number (b) Bipartite graph

(c) Maximal matching (d) $M$ - alternating path.

--------END OF PAPER--------