GANPAT UNIVERSITY
M.Sc. **First/~~Third~~** Semester Examination (C.B.C.S) Nov-Dec, 2013
Subject: Mathematics
Paper: MMA 101 CAN Complex Analysis

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1(a) State and prove triangle inequality for complex numbers. When does the equality hold? [07]

(b) Derive Cauchy-Riemann equations for a differentiable function. Show that the function $f(z) = z^2$ satisfies C.R. equations at every point. [07]

Q-2(a) Show that an analytic function $f(z)$ on a domain D reduces to a constant provided modulus of $f(z)$ is constant on D. [07]

(b) Define harmonic function and harmonic conjugate. Prove that $f = u + iv$ is analytic on a domain D iff v is a harmonic conjugate of u on D. [07]

Q-3(a) Suppose f is continuous on a domain D and the integral is independent of the path. Then show that f has an antiderivative on D. [07]

(b) State and prove Cauchy's theorem. Is the converse true? Justify it. [07]

Q-4 Write answer of following questions.

1. Let f be continuous at z_0 and $f(z_0) \neq 0$. Then there exist a neighborhood of z_0 on which f will be different from zero. [03]

2. Evaluate $(-8i)^{\frac{1}{3}}$. Also find out the principle root. [04]

SECTION: II

Q-5(a) State and prove the fundamental theorem of algebra. [07]

(b) Is $\cos z$ bounded? Justify your answer. State and prove the result used here. [07]

Q-6(a) Explain types of singularity with an example. [07]

(b) State and prove Cauchy's integral formula and explain its meaning and hence evaluate $\int \frac{e^{2z}}{(z-1)(z-2)}\, dz$ over the contour $|z| = 3$. [07]

Q-7(a) State and prove Maximum Modulus principle. Illustrate it by an example. [07]

(b) Evaluate $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)^2}\, dx$. [07]

Q-8 Write answer of following questions.

1. Evaluate $\int \frac{e^{-z}}{(z-1)^2}\, dz$ over the contour $|z| = 2$. State the result used here. [03]

2. Find out the Laurent's series expansion of $\frac{-1}{(z-1)(z-2)}$, where $1 < |z| < 2$. [04]

--------END OF PAPER--------

GANPUT UNIVERSITY

M.Sc. First Semester Examination (C.B.C.S.) Nov.- Dec., 2013

Subject: Mathematics

Paper: MMA 102 ALB Algebra

Time: 3 hours Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book

Section -1

Q-1 (a) Let N be a subgroup of the group G then prove that following statements are equivalents
1. N is a normal in G
2. $gNg^{-1} = N$ for every $g \in G$ 7
3. $gN = Ng$ for every $g \in G$

(b) Let Φ be a onto homomorphism from G to G' with kernel K and N' be a normal subgroup of G', $N = \{ x \in G / \Phi(x) \in N' \}$. Then prove that $\frac{G}{N} \approx \frac{G'}{N'}$ equivalently $\frac{G}{N} \approx \left(\frac{G}{K}\right) / \left(\frac{N}{K}\right)$ 7

Q-2 (a) If p is a prime number and G is a finite abelian group, p divides O(G), then G has an element of order p. 7

(b) State and prove langrage theorem. Also prove that if a is an element of finite group G, then O(a) divides O(G). 7

Q-3 (a) Prove that S_{p^k} has a p – sylow subgroup. 7

(b) Prove that two abelian groups of order p^n are isomorphic if and only if they have same invariant. 7

Q-4 (a) Prove that $n(k) = 1 + p + p^2 + \cdots + p^{k-1}$. 3

(b) Prove that H is a subgroup of G if and only if for $a, b \in H$, $ab^{-1} \in H$ 4

Section -2

Q-5 (a) Define the field of quotients of an integral domain. Prove that every integral domain can be imbedded in a field. 7

(b) Define Euclidean ring and prove that Euclidean ring is a principle ideal ring. 7

Q-6 (a) State and prove the unique factorization theorem for polynomial ring. 7

(b) Prove that R[x] is an integral domain. 7

Q-7 (a) Let Φ be an onto homomorphism from R to R' with kernel I, then prove that $\frac{R}{I} \approx R'$ 7

(b) Prove that every element in Euclidean ring R is either unit in R or can be written as the product of the finite number of prime element in R. 7

Q-8 (a) Find the greatest common divisor of the polynomials $x^2 + 1$ and $x^6 + x^3 + x + 1$. 3

(b) Let p be a prime integer of the form $4n + 1$, then $p = a^2 + b^2$ for some integer a and b. 4

-----END OF PAPER -----

GANPAT UNIVERSITY

M.Sc. First Semester Examination (C.B.C.S) Nov-Dec. 2013

Subject: Mathematics

Paper: MMA 103 MDE Methods of Differential Equations

Time: 3 hours — Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1 (a) Solve $y'' + xy' + y = 0$ near 0. [07]

(b) Solve by using the method of variation of parameters. [07]

(i) $y'' + y = cosec\, x$

(ii) $y'' - 2y' + y = 2x$

Q-2 (a) State and prove Rodrigue's formula for the Legendre polynomial and hence express $x^2 - 3x + 1$ in terms of Legendre polynomials. [07]

(b) Prove that $J_n(x + y) = \sum_{k=-\infty}^{\infty} J_{n-k}(x) J_k(y)$ and hence deduce that [07]

$1 = {J_0}^2(x) + 2\sum_{k=1}^{\infty} {J_k}^2(x)$.

Q-3 (a) Prove that $nP_n(x) = (2n-1)x\, P_{n-1}(x) - (n-1)P_{n-2}(x)$; $n \geq 2$. [07]

(b) Prove that $J_m(x) = \frac{1}{\pi} \int_0^{\pi} \cos(m\theta - x\sin\theta)\ d\theta$ and hence deduce that [07]

$J_0(x) = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos(x\sin\theta)\ d\theta$.

Q-4. Write answer of following questions.

1. Find the Fourier Legendre expansion of a function defined by [03]

$$f(x) = \begin{cases} 0\ ; & -1 \ll x < 0 \\ x; & 0 \ll x \ll 1 \end{cases}$$

2. Prove that $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$. $(n \geq 1)$ [04]

SECTION: II

Q-5. State and solve Bessel's differential equation of order p near 0. [14]

Q-6 (a) Derive an integral representation for the Gauss Hyper geometric function. [07]

(b) Find the third approximation of the solution of the equation $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^2z + x^4y$ by Picard's method, $y = 5$ and $z = 1$ when $x = 0$. [07]

Q-7 (a) Determine which of the following equations are integrable and find the solution of those which are integrable. [07]

(i) $(y^2 + xz)dx + (x^2 + yz)dy + 3z^2dz = 0$.

(ii) $y(1 + z^2)dx - x(1 + z^2)dy + (x^2 + y^2)dz = 0$

(b) Solve $2(z + xp + yq) = yp^2$ by using Charpit's method. [07]

Q-8. Write answer of following questions.

1. Prove that between two positive zeros of $J_1(x)$ there is a zero of $J_0(x)$. [03]

2. Prove that $\frac{d}{dx} F(\alpha, \beta; \gamma; x) = \frac{\alpha\beta}{\gamma} F(\alpha + 1, \beta + 1; \gamma + 1; x)$. [04]

--------END OF PAPER--------

GANPAT UNIVERSITY

M.Sc. **First** Semester Examination (C.B.C.S) Nov-Dec, 2013

Subject: Mathematics

Paper: 104: Operations Research

Time: 3 hours | Total Marks: 70

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer books.

SECTION: I

Q-1 (a) Describe the steps in dual simplex method of solving an LPP. 07

(b) Find the maximum value of $z = 107x_1 + x_2 + 2x_3$ 07

subject to

$14x_1 + x_2 - 6x_3 + 3x_4 = 7, 16x_1 + x_2 - 6x_3 \leq 5,$

$3x_1 - x_2 - x_3 \leq 0 \ , x_1, x_2, x_3, x_4 \geq 0$

Q-2 (a) Describe Vogel's approximation method to obtain initial basic feasible solution to a transportation problem. 07

(b) Solve the following assignment problem to maximize the profit. 07

	D1	D2	D3	D4
O1	3	4	11	9
O2	5	7	8	9
O3	5	6	6	7
O4	4	6	8	8

Q-3 (a) Describe steps in Hungarian method for optimization of an assignment problem. 07

(b) Obtain optimal strategies for both players and the value of the game whose payoff matrix is given by 07

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

Q-4 Write answer of following questions.

1. Describe the Maximin-Minimax principle of game theory. 03
2. M/S MD Ambani Refineries use two type of crude oils crude-A and crude-B to produce petrol and diesel. The production is done by two different processes. The data is given below 04

Process	Input		Output	
	Crude-A	Crude-B	Petrol	Diesel
1	6	4	6	9
2	5	6	5	5

The maximum amount of crude-A and crude-B is 250 units and 200 units respectively. The market survey says that at least 150 units of petrol and 130 units of diesel must be produced. The profit per production from process-1 and process-2 is Rs 4 and Rs. 5 respectively. Formulate the problem for maximum profit.

SECTION: II

Q-5 (a) What is meant by dominance property. Describe general rules for dominance. 07

(b) Solve the following game using Algebraic method 07

PLAYER A	PLAYER B		
	B_1	B_2	B_3
A_1	10	5	-2
A_2	13	12	15
A_3	16	14	10

Q-6 (a) Using the graphical Method, calculate the total elapsed time needed to complete both jobs on five machines. 07

MACHINES

Job						
Job 1	Sequence	A	B	C	D	E
	Time(hours)	6	8	4	12	4
Job 2	Sequence	B	C	A	D	E
	Time(hours)	10	8	6	4	12

(b) Define: Event, Network, Critical Path, Forward Pass, Backward Pass and Float. 07

Q-7 (a) Explain Johnson's algorithm for n jobs through three machines. 07

(b)Find the sequence that minimized the total elapsed time required to complete the following Books on printing machine and binding machine. 07

Books	1	2	3	4	5	6	7
Printing time(hours)	20	90	80	20	120	15	65
Binding time(hours)	25	60	75	30	90	35	50

Q-8 Write answer of following questions.

1.Distinguish between CPM and PERT 03

2. Draw a network diagram of activities for the project. 04

ACTIVITY	DESCRIPTION	PREDESSOR ACTIVITY
A	Open work order	----
B	Get material for X	A
C	Get material for Y	A
D	Turn X on lathe	B
E	Turn Y on lathe	B, C
F	Police Y	E
G	Assembly X and Y	D, F
H	Pack	G

--------END OF PAPER--------

GANPAT UNIVERSITY

M.Sc. First Semester Examination (C.B.C.S) Nov-Dec, 2013

Subject: Mathematics

Paper: MMB 105 GTH Graph Theory

Total Marks: 70

Time: 3 hours

Instructions:

1. Attempt any three questions from each section, of which question No. 4 and 8 are compulsory
2. Answer each section in separate answer book.

SECTION: I

Q-1(a) Let G be a non-empty graph with at least two vertices. Then prove that G is bipartite if and only if G has no odd cycles. 07

(b) Let G be a graph with n vertices. Then following statements are equivalents. (i)G is a tree. (ii)G is an acyclic graph with n-1 edges. (iii) G is a connected graph with n-1 edges. 07

Q-2(a) A connected graph G is Euler iff the degree of every vertex is even 07

(b) Define with example of Arborescence digraph. Also prove that Arborescence is a tree in which every vertex other than the root has an indegree of exactly one. 07

Q-3(a) How long a longest circular sequence of 1's and 0's is such that no subsequence of r bits appears more than once in the sequence? Give the suitable example. 07

(b) If G is a simple graph with n vertices, $n \geq 3$ and the degree $d(v) \geq n/2$, for every vertex v of G, then G is Hamiltonian. 07

Q-4. Write answer of following questions.

1.Define the graph with an example. 03

Isomorphic graphs, Euler graph, Hamiltonian graph.

2.Show that an edge e in graph G is bridge iff e is not part of any cycle in G. 04

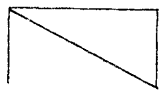
SECTION: II

Q-5(a) Let B and A are the circuit matrix and incidence matrix of a self -loop free digraph such that the columns in B and A are arranged using the same order of edges. Then $AB^T = BA^T = 0$ 07

(b) A simple digraph G of n vertices and n – 1 directed edge is an arborescence rooted at v_1 iff the cofactor K_{11} of K (G) is equal to 1. 07

Q-6(a) Define with an example of proper coloring and chromatic number. Also prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length. 07

(b) Define Chromatic polynomial. Determine chromatic polynomial of the following graph 07



Q-7(a) State and prove the Hall's Marriage Theorem. 07

(b) Let G be a simple graph and m be a matching in G. Then m is a maximum matching in G if and only if there is no m-augmenting path in G. 07

Q-8. Write answer of following questions.

1.Let G be a simple graph with n vertices. If G is k- chromatic then $\beta(G) \geq n/k$ 03

2.Define with an example. 04

In-tree, Fundamental circuit and Fundamental cut set.

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