MATHEMATICS

Paper 2: Solid Geometry, Abstract Algebra and Real Analysis

Time: 3 hours Max.Marks:80

SECTION - A

Answer ALL questions. 4 x 15 = 60

1. a) i) Show that every finite integral domain is a field. (8marks)

ii) Show that the characteristic of an integral domain is either zero or prime. (7 marks)

Or

b) i) Show that the ring of integers is a principal ideal ring. (8marks)

ii) If f is a homomorphism of a ring R into the ring R^1 then show that '**f**' is an into isomorphism if and only if ker **f** $=\{0\}$ (7 marks)

2. a) i) If f: [a, b] $\rightarrow$ R is continuous on [a, b] then show that f is Uniformly continuous on [a, b]. (8marks)

ii) Prove that $\frac{\pi}{6}+\frac{\sqrt{3}}{15}<\sin(0.6)<\frac{\pi}{6}+\frac{1}{8}$ (7 marks)

Or

b) i) State and prove Cauchy's mean value theorem. (8marks)

ii) Determine the constants a and b so that the function defined by $f(x)=2x+1$ if $x\le 1$, ax^2+b if $1<x<3$, $5x+2a$ if $x\ge 3$ is continuous everywhere (7 marks)

3. a) i) If f: [a,b] $\rightarrow$ R is monotonic on [a, b] then show that f is integrable on [a,b] (8marks)

ii) If f(x) = x^3 is defined on [0,a] show that f $\in$ R([0,a]) and $\int_0^a f(x)\,dx=\frac{a^4}{4}$ (7 marks)

Or

b) i) State and prove fundamental theorem of integral calculus. (8marks)

ii) Prove that $\frac{1}{\pi}\le\int_0^1\frac{\sin\pi x}{1+x^2}\,dx\le\frac{2}{\pi}$. (7 marks)

4. a) i) Find the S.D. between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{34}$. Find also the equations and the points in which the S.D. line meets the given lines. (8 marks)

ii) A Variable plane is at a constant distance 3p from the origin and meets the coordinate axes in A, B, C. Show that the locus of the centroid of the ΔABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$ ` (7 marks)

Or

b) i) Show that the plane 14x -8y+ 13 = 0 bisects the obtuse angle between the planes 3x + 4y -5z +1 = 0& 5x + 12y – 13z = 0 (8 marks)

ii) Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at (1, -2, 1) and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$. (7 marks)

SECTION – B

Answer any FOUR Questions 4x5=20

5. Show that a field has no proper ideals.
6. If f is a homomorphism of a ring R in to a ring R' then prove that kernel of **f** is an ideal of R.
7. Examine the continuity of the function **f** defined by $f(x) = \lim_{n\to\infty} \frac{x^n}{1+x^n e^x} \quad \forall x \geq 0$
8. Show that the function f(x) = x sin (1 / x) if x≠0, f(x) = 0 if x = 0 is continuous at x = 0 but not differentiable at x = 0
9. If $f(x)=x^2$ on [0,1] and $p=\left\{0,\frac{1}{4},\frac{2}{4},\frac{3}{4},1\right\}$ then compute $L(p,f)$ and $U(p,f)$.
10. State and prove first mean value theorem of integral calculus.
11. Find the image of the point (2, -1, 3) in the plane $3x-2y+z=9$.
12. Find the limiting points of the coaxial system defined by the Spheres $x^2+y^2+z^2+4x+2y+2z+6=0$ and $x^2+y^2+z^2+2x-4y-2z+6=0$