MATHEMATICS

(Common for B.Sc. / BA)

Paper 3: Linear Algebra and Vector Calculus

Time: 3 hours Max Marks: 80

SECTION - A

Answer all the FOUR questions. Each question carries 15 marks. 4X15=60

1. a) i) State and Prove the necessary and sufficient condition for a non-empty subset of a vector space to be a subspace. 7M
   ii) State and Prove Rank and Nullity theorem of Linear Transformation. 8M

   (or)

   b) i) Prove that $\dim(w_1 + w_2) = \dim w_1 + \dim w_2 - \dim(w_1 \cap w_2)$ 8M

   ii) T: $R^3 \to R^2$, the matrix of T = $\begin{bmatrix} 1 & 2 & -3 \\ 4 & 2 & -1 \end{bmatrix}$ Determine the transformation relative to bases { (1,2,1), (2,0,1), (0,3,4) } and { (2,1), (0,5) } 7M

2. a) i) Prove that dim(V/W) = dimV – dimW, where W is a subspace of a vector space V(F). 8M
   ii) Show that the transformation T: $R^3 \to R^3$, defined by T(x,y,z) = (x – y , 0, y +z) is a linear transformation. 7M

   (or)

   b) i) State and Prove necessary and sufficient condition the vector space to be the direct sum of its two subspaces w_1 and w_2. 7M
   ii) Prove that two finite dimensional vector spaces U(F) and V(F) are isomorphic if and only if dimU = dimV.

   8M

3. a) i) State and Prove Bessel's inequality.. 7M
   ii) Find the characteristic values and characteristic vectors of the matrix 8M

   $$A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 4 \end{pmatrix}$$

   (or)

   b) i) State and Prove Cayley-Hamilton Theorem for matrices. 8M

   ii) Using Gram-Schmidt Orhogonalisation Process to find and orthonormal basis of the vector space $R^3(R)$ from the basis B = { (1, -1, 2), (0, 2, 1), (1, 2, 0) }. 7M

4. a) i) State and Prove Green's theorem in a plane 8M
   ii) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $z = x^2 + y^2 - 3$ at (2, -1, 2) 7M

   (or)