[05 – 2102]

II/IV B.E. DEGREE EXAMINATION.

First Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

MATHEMATICS — III

(Common for all branches Except Chemical Engineering and Bio-Technology Common MSEEE)

(w.e.f. the admitted batch of 2004-2005 and after batches)

Time : Three hours | Maximum : 70 marks

Answer Part A and Four questions from Part B of Seven questions.

All questions carry equal marks.

Questions of Part A must be answered at one place.

PART A

1. (a) If $\overline{A}$ is a constant vector and $\overline{R} = x\,i + y\,j + z\,k$ then find $div\,(\overline{A} \times \overline{R})$.

   (b) Find the directional derivative of $2x^2y^2 + 5z$ at $(-1, 1, 2)$ in the direction $3\,i - 2j - k$.

   (c) What is meant by flux?

(d) Form the partial differential equation by eliminating the arbitrary function from $z = y f(x) + x g(y)$.

(e) Write down the assumptions for one dimensional wave equation.

(f) Write down the Fourier sine and Fourier cosine integrals.

(g) Write the relation between Fourier and Laplace transforms.

PART B

2. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(b) Prove that

$$div(\overline{A} \times \overline{B}) = \overline{B} \cdot curl\ \overline{A} - \overline{A} \cdot curl\ \overline{B}.$$

3. (a) Evaluate $\iint_S \overline{F} \cdot \overline{n}\, dS$ where

$\overline{F} = z i + x j - 3y^2 z k$ where S is the surface of the cylinder $x^2 + y^2 = 16$ in the first octant between $z = 0$ and $z = 5$.

(b) Using Green's theorem show that

$$\int_c (x y^2\, dy - x^2 y\, dx = \frac{35}{16} \pi a^4 \text{ where}$$

$C : r = a(1 - \cos\theta)$.

4. (a) Express the vector field $2yi - z\,j + 3x\,k$ in spherical polar coordinate system.

(b) Solve $y\,z\,p + z\,x\,q = x\,y$.

5. (a) Solve $(D^2 + 3DD' + 2\,D'^2)\,z = x + y$.

(b) Solve $(D^2 - DD' + D' - 1)\,z = \cos\,(x + 2\,y)$.

6. An insulated rod of length L has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 20°C and maintained at 20°C, find the temperature at a distance x from A at time t.

7. (a) Express the function $f\,(x) = \begin{cases} 1 \text{ for } |x| \le 1, \\ 0 \text{ for } |x| > 1, \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin \lambda \cos \lambda\, x}{\lambda}\, d\,\lambda$.

(b) Find the finite Fourier sine transform of $f\,(x) = 2x,\ 0 < x < 4$.

8. (a) Find the Fourier cosine transform of e^{-x^2}.

(b) Using Parseval's identity show that $\int_0^\infty \frac{dt}{(t^2 + 1)^2} = \frac{\pi}{4}$.
