[05 – 2115]

II/IV B.E. DEGREE EXAMINATION.

First Semester

Electronics and Communication Engineering

MATHEMATICS — III

(Effective from the Admitted Batch of 2013–2014)

Time : Three hours | Maximum : 70 marks

Part A is compulsory.

Answer any FOUR from Part B.

All questions carry equal marks.

PART A

1. (a) Define Circulation.

   (b) If $\bar{f}(t)$ has a constant magnitude then prove that $\bar{f} \cdot \frac{d\bar{f}}{dt} = 0$.

   (c) Eliminate f from $z = x + y + f(xy)$.

   (d) Solve : $(D^2 - D'^2 - 3D + 3D')z = 0$.

8. (a) Solve the integral equation $\int_0^\infty f(x)\cos\lambda x \, dx = e^{-\lambda}$ and hence show that $\int_0^\infty \frac{\cos\lambda x}{1+x^2} dx = \frac{\pi}{2} e^{-\lambda}$.

(b) Using Parseval's identity, show that $\int_0^\infty \frac{dt}{(t^2+1)^2} = \frac{\pi}{4}$.

(e) Distinguish between steady and unsteady states condition in one dimensional heat flow equation.

(f) Find the finite Fourier cosine transform of $f(x) = 2$, $0 < x < \pi$.

(g) State the convolution theorem for Fourier transform.

PART B

2. (a) If r and $\overline{R}$ have their usual meanings and $\overline{A}$ is a constant vector, then prove that $\nabla \times \left(\frac{\overline{A} \times \overline{R}}{r^n} \right) = \frac{2-n}{r^n} \overline{A} + \frac{n(\overline{A} \cdot \overline{R})}{r^{n+2}} \overline{R}$.

(b) Find the values of a, b and c so that the directional derivative of $f = axy^2 + byz + cz^3x^3$ at (1, 2, −1) has a maximum magnitude 64 in a direction parallel to the Z-axis.

3. (a) Use Green's theorem to evaluate $\oint_c (x^2 + xy)\,dx + (x^2 + y^2)\,dy$, where c is the square formed by the lines $x = \pm 1$, $y = \pm 1$.

(b) Use divergence theorem to evaluate $\iint_S (x\,dy\,dz + y\,dx\,dz + z\,dx\,dy)$, where S is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant.

4. (a) A fluid motion

$$V = (y \sin z - \sin x) i + (z \sin z + 2yz) j + (xy \cos z + y^2) k .$$

Is the motion irrotational? If so find the velocity potential.

(b) Solve : $\left(\frac{b-c}{a}\right) yzp + \left(\frac{c-a}{b}\right) xzq = \left(\frac{a-b}{c}\right) xy .$

5. (a) Solve : $9(p^2 z + q^2) = 4 .$

(b) Solve : $(D^2 - 2DD' + D'^2) z = x^2 y^2 e^{x+y} .$

6. (a) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0, t) = u(10, t) = 0$ for all t; $u(x, 0) = 0$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 3 \sin 2\pi x - 2 \sin 5\pi x .$

(b) Derive the solution of Laplace's equation in polar coordinates.

7. (a) Express the function $f(x) = \begin{cases} 1, & \text{for } |x| \le 1 \\ 0, & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} \, d\lambda .$

(b) Find the Fourier sine transform of $\frac{e^{-ax}}{x} .$