

[07 – 2113]

II/IV B.Tech. DEGREE EXAMINATION

Computer Science and Engineering

First Semester

DISCRETE MATHEMATICAL STRUCTURES – I

(Common with Information Technology and  
Dual Degree Program in SE and IT)

(Effective from the admitted batch of 2006–2007)

Time : Three hours \*

Maximum : 70 marks

Questions No. 1 is compulsory.

Answer any FOUR from the remaining.

All questions carry equal marks.

Answer all parts of any question at one place.

1. Answer the following :

- (a) What is logical implication? Give an example.
- (b) Define the basic counting principles.
- (c) How many 5-card hands consist only of hearts?
- (d) Write the concept of graph isomorphism.

- (e) Define linear recurrence relation and when it becomes homogeneous recurrence relation.
- (f) Explain the concept of cut edge and cut vertex with an example.
- (g) Define binary search tree.
2. (a) Prove that  $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$  is a tautology.
- (b) State and prove the pigeonhole principle.
3. (a) Prove the validity of the following arguments :

*Ever living thing is a plant or an animal.*

*David's dog is alive and it is not a plant.*

*All animals have hearts.*

*Hence David's dog has heart.*

- (b) Prove that if  $F_n$  is the  $n^{\text{th}}$  Fibonacci number, then

$$F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1 + \sqrt{5}}{2} \right)^{n+1} - \left( \frac{1 - \sqrt{5}}{2} \right)^{n+1} \right]$$

for all integers  $n \geq 0$ .

4. (a) Describe the principle of inclusion-exclusion with an illustrative example.  
(b) Complete the coefficients of

$$\sum_{r=0}^{\infty} d_r X^r = \frac{X^2 - 5X + 3}{X^4 - 5X^2 + 4}.$$

5. (a) Solve the recurrence relation  $a_n - 7a_{n-1} + 10a_{n-2} = 0$  for  $n \geq 2$ .

- (b) Find a particular solution to

$$a_n - 7a_{n-1} + 10a_{n-2} = 7.3^n \text{ for } n \geq 2.$$

6. (a) If  $A = (V, E)$  is a digraph, then prove that for  $n \geq 1, (x, y) \in E^n$  iff there is a directed path of length  $n$  from  $x$  to  $y$  in  $A$ .

- (b) Explain Topological sort algorithm with an example.

7. (a) Prove that a tree with  $n$  vertices has exactly  $n - 1$  edges.

- (b) Write BFS and DFS algorithms.

8. (a) Prove that a complete graph  $K_n$  is planar iff  $n \leq 4$ .

- (b) Prove that every simple planar graph is 5-colorable.