

[07 - 2113]

II/IV B.Tech. DEGREE EXAMINATION.

First Semester

Computer Science and Engineering

DISCRETE MATHEMATICAL STRUCTURES - I

(Common with IT)

(Effective from the admitted batch of 2006-2007)

Time : Three hours

Maximum : 70 marks

First question is compulsory.

Answer any FOUR from the remaining questions.

All questions carry equal marks.

Answer all Parts of any question at one place.

1. Answer the following.

- (a) Define Power set of a given set. Find the power set of A where $A = \{a, b, c\}$.
- (b) Write the rules of inference for quantified propositions.
- (c) How many committees of 5 or more can be chosen from 9 people?

- (d) A certain computer center employs 100 computer programmers. Of these 47 can program in FORTRAN, 35 in pascal and 23 can program both languages. How many can program in neither of these 2 languages.
- (e) Find the coefficient of X^{10} in $(1 + X + X^2 + \dots)^2$.
- (f) Find the transitive closure of R if $R = \{(a, b), (b, c), (c, d), (d, e)\}$.
- (g) Write the expression $(a + 5)((3b + c)/(d + 2))$ as a tree structure.
2. (a) Show that $((P \rightarrow (q \vee r) \wedge (\sim q)) \rightarrow (p \rightarrow r))$ is a tautology.
- (b) Symbolize the following argument and check for its validity :

All men are fallible

All kings are men

Therefore, all kings are fallible.

3. (a) Find and prove a formula for the sum of the first n cubes, that is, $1^3 + 2^3 + \dots + n^3$.
- (b) Enumerate the number of non negative integral solutions to the equality $x_1 + x_2 + x_3 + x_4 + x_5 \leq 19$.

4. (a) How many ways can the letters $\{4.a, 3.b, 2.c\}$ be arranged so that all the letters of the same kind are not in a single block?

(b) Calculate $B(X) = \sum_{r=0}^{\infty} b_r X^r = \frac{1}{X^2 - 5x + 6}$.

5. (a) Solve the recurrence relation

$$a_n - 7a_{n-1} + 10a_{n-2} = 0 \text{ for } n \geq 2 \text{ by generating functions.}$$

- (b) Solve the recurrence relation

$$a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = n \text{ for a particular solution.}$$

6. (a) Show that the relation $Q = \{(f, g) / f : N \rightarrow R, g : N \rightarrow R, f \text{ is in } O(g)\}$ is reflexive and transitive, but not a partial ordering.

- (b) Prove that if $A = (V, E)$ is a digraph, then for $n \geq 1, (x, y) \in E^n$ iff there is a directed path of length n from x to y .

7. (a) Prove that a tree with n vertices has exactly $n - 1$ edges.
- (b) Write BFS and DFS for a spanning tree.
8. (a) If G is a connected plane graph, then prove that $|V| - |E| + |R| = 2$.
- (b) Prove that every simple planar graph is 5-colorable.