[07 – 2113]

II/IV B.Tech. DEGREE EXAMINATION

Computer Science and Engineering

First Semester

DISCRETE MATHEMATICAL STRUCTURES – I

(Common with Information Technology and Dual Degree Program in SE and IT)

(Effective from the admitted batch of 2006–2007)

Time : Three hours   Maximum : 70 marks

Questions No. 1 is compulsory.

Answer any FOUR from the remaining.

All questions carry equal marks.

Answer all parts of any question at one place.

1. Answer the following :

   (a) What is logical implication? Give an example.

   (b) Define the basic counting principles.

   (c) How many 5-card hands consist only of hearts?

   (d) Write the concept of graph isomorphism.

(e) Define linear recurrence relation and when it becomes homogeneous recurrence relation.

(f) Explain the concept of cut edge and cut vertex with an example.

(g) Define binary search tree.

2. (a) Prove that $((p \to q) \wedge (q \to r)) \to ((p \vee q) \to r)$ is a tautology.

(b) State and prove the pigeonhole principle.

3. (a) Prove the validity of the following arguments :

Ever living thing is a plant or an animal.

David's dog is alive and it is not a plant.

All animals have hearts.

Hence David's dog has heart.

(b) Prove that if F_n is the n^{th} Fibonacci number, then

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$$

for all integers $n \geq 0$.

4. (a) Describe the principle of inclusion-exclusion with an illustrative example.

(b) Complete the coefficients of

$$\sum_{r=0}^{\infty} d_r X^r = \frac{X^2 - 5X + 3}{X^4 - 5X^2 + 4}.$$

5. (a) Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n \geq 2$.

(b) Find a particular solution to

$a_n - 7a_{n-1} + 10a_{n-2} = 7.3^n$ for $n \geq 2$.

6. (a) If $A = (V, E)$ is a digraph, then prove that for $n \geq 1, (x, y) \in E^n$ iff there is a directed path of length n from x to y in A.

(b) Explain Topological sort algorithm with an example.

7. (a) Prove that a tree with n vertices has exactly $n-1$ edges.

(b) Write BFS and DFS algorithms.

8. (a) Prove that a complete graph K_n is planar iff $n \leq 4$.

(b) Prove that every simple planar graph is 5–colorable.
