$1^2 \quad 2^2 \quad 3^2 \qquad 6$

MATHEMATICS – 1 (2011)

Part A is compulsory.
Answer any FOUR questions from Part B.
Each question will carry 14 marks.

PART - A

1. (a) If $u = lx + my, v = mx - ly$, show that $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v = \frac{l^2}{l^2+m^2}$.

   (b) Evaluate $\iint xy\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

   (c) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2\, yz\, dx\, dy\, dz$.

   (d) Test for convergence $\sum \frac{(n+1)(\quad)n+2}{n^2\sqrt{n}}$.

   (e) Discuss the convergence of the series $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \cdots \infty$.

   (f) Find the value of b^n in the Fourier series of the expansion $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

   (g) Obtain the sine series for $f(x) = x$ in the interval $0 \le x \le \pi$.

PART – B

2. (a) If $u = \tan^{-1}\left(y^2/x\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y\frac{\partial^2 u}{\partial y^2} = sin^2 u \cdot \sin 2u$.

   (b) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

3. (a) Change the order of integration and evaluate $I = \int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$.

   (b) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

4. (a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the hyperboloid $x^2 + y^2 - z^2 = 1$.

   (b) Prove that $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$.

5. (a) Show that the line $\frac{x+1}{3} + \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z = 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. Find their point of intersection and the plane in which they lie.

   (b) Prove that the planes 2x + 3y + 4z = 6, 3x +4y + 5z = 20, x + 2y + 3z = 0 form a prism.

6. (a) Show that the spheres $x^2 + y^2 + z^2 + 6y + 14z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ intersect at right angles. Find their plane of intersection.

   (b) Find the equation of the cone whose vertex is (3,4,5) and base is the conic $3y^2 + 4z^2 = 16$, $z + 2x = 0$.

7. (a) State the values of x for which the series $\frac{1}{1-x} + \frac{1}{2(1-x)^2} + \frac{1}{3(1-x)^3} + \cdots \infty$ converge.

   (b) Test the series for convergence $1 + \frac{3}{7}x + \frac{3\cdot 6}{7\cdot 10}x^2 + \frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3 + \frac{3\cdot 6\cdot 9\cdot 12}{7\cdot 10\cdot 13\cdot 16}x^4 + \cdots \infty$.

8. (a) Obtain Fourier series for the function $f(x) = \pi x, 0 \le x \le 1$ and $f(x) = \pi(2-x), 1 \le x \le 2$.

   Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

   (b) Obtain the Fourier expansion of $x\,sinx$ as a cosine series in $(0, \pi)$. Hence show that $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} - \frac{1}{5\cdot 7} - \cdots = \frac{\pi-2}{4}$.