MATHEMATICS – 1 (2012)

Part - A is compulsory.
Answer any FOUR questions from Part B.
Each question will carry 14 marks.

PART - A

1. (a) If an error of 1% is made in measuring its length and breadth, then find the percentage error in the area of a rectangle.

   (b) Evaluate $\iint dx\,dy$ over the area bounded by x = 0, y = 0, $x^2 + y^2 = 1$ and 5y = 3.

   (c) Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 z\,dx\,dy\,dz$.

   (d) Test the series $\frac{2^p}{1^q} + \frac{3^p}{2^q} + \frac{4^p}{3^q} + \cdots \infty$.

   (e) Find what values of x the series $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$ converges uniformly.

   (f) State Dirichlet conditions for the expansion of a function as a Fourier series in the interval $C_1 \leq X \leq C_2$.

   (g) Find the half-range series for 1 in $(0, \pi)$.

PART - B

2. (a) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, show that $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2} = -4(x+y)^{-2}$.

   (b) The quantity Q of water flowing over a notch is given by $Q = \frac{8}{15} \times 0.64 \times \sqrt{2g}H^{5/2}$, where H is given the head of the notch. What is the percentage error in Q caused by measuring H as 0.198 instead of 0.2 ?

3. (a) Show that $\iint_R r^2 \sin\theta\,dr\,d\theta = \frac{2a^3}{3}$, where R is the region bounded by the semi circle $r = 2a\cos\theta$, above the initial line.

   (b) Find the area enclosed b y the curves $y = \frac{3x}{x^2+2}$ and $4y = x^2$.

4. (a) Find, by triple integration, the volume in the positive octant bounded by the coordinate planes and the plane x+2y+3z = 4.

   (b) Prove that $\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}}dx \times \int_0^{\infty} x^2 e^{-x^4}dx = \frac{\pi}{4\sqrt{2}}$.

5. (a) Find the equation of the plane through the points $(-1,1,1)$ and $(1,-1,1)$ and perpendicular to the plane $x+2y+2z = 5$.

   (b) Find the equation of the plane containing the line $\frac{x}{2} = \frac{y}{-1} = \frac{z+1}{-2}$ and parallel to the line joining the points $(-1,1.-2)$ and $(3,-2,4)$.

6. (a) Find the equation of the sphere through the four points $(1,2,3), (0,-2,4), (4,-4,2)$ and $(3,1,4)$.

   (b) Find the angle between the lines of section of the plane $3x+y+5z = 0$ and cone $6yz - 2zx+5xy = 0$.

7. (a) Discuss the convergence of the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\cdots+\frac{x^n}{n^2+1}+\cdots\infty$.

   (b) Examine the series $1+\frac{2^2}{3^2}+\frac{2^2.4^2}{3^2.5^2}+\frac{2^2.4^2.6^2}{3^2.5^2.7^2}+\cdots\infty$.

8 (a) Find the Fourier expansion for $f(x) = \pi x$ from $x = -C$ to $x = C$.

   (b) Find the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$. Hence show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots = \frac{\pi^2}{6}$.