MATHEMATICS – II (2011)

Part A is compulsory.
Answer any FOUR questions from Part B.
Each question will carry 14 marks.

PART – A

1. (a) Using Cayley-Hamilton theorem, find A^8 if $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

(b) Show that $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & u \end{bmatrix}$ is a Hermitian matrix.

(c) Find the solution of $\frac{dy}{dx} = e^{2x-3y} + 4x^2 e^{=3y}$.

(d) Solve $p = \sin(y - xp)$.

(e) Solve $4\,y''' + 4\,y'' + y' = 0$.

(f) Find $L\left[\frac{\sin at}{t}\right]$.

(g) Find $L^{-1}\left[\frac{s}{(s^2=1)^2}\right]$.

PART B

2. (a) Reduce the matrix $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ to normal form and find its rank.

   (b) Show that if $\lambda \neq 5$, the system of equations $3x - y + 4z = 3, x + ry - 3z = -2, 6x + 5y + \lambda z = -3$ have a unique solution. If $\lambda = -5$, show that the equations are consistent. Find the solutions in each case.

3. (a) Find the characteristic equation of the matrix,

   $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1}. Also find the matrix represented by

   $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

   (b) Reduce the quadratic form

   $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ into sum of squared by an orthogonal transformation and give the matrix of transformation.

4. (a) Solve $e^y \left[\frac{dy}{dx} + 1\right] = e^x$.

   (b) Solve $\left[x\, y^2 - e^{1/x^3}\right] dx - x^2 y\, dy = 0$.

5. (a) Solve $(D^2 + 4D + 3)y = e^{-x} \sin x + x\, e^{3x}$.

   (b) An emf $E \sin pt$ is applied at $t = 0$ to a circuit containing capacitance C and inductance L. The current I satisfies the equation

   $L \frac{di}{dt} + \frac{1}{C} \int i\, dt = E \sin pt$. If $p^2 = \frac{1}{LC}$ and initially the current I and the charge q are zero, show that the current at time t is $\left[\frac{Et}{2L}\right] \sin pt$, where $i = \frac{dq}{dt}$.

6. (a) Solve in series $(1+x)^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$.

   (b) Show that $(2n+1)p_n(x) = p'_{n+1}(x) - p'_{n-1}(x)$.

7. (a) Evaluate $L\left[\int_0^t \frac{e^t \sin t}{t}\, dt\right]$.

   (b) Using convolution theorem evaluate

   $L^{-1}\left[\frac{1}{(s-2)(s+2)^2}\right]$.

8. (a) Solve by transform method

   $ty'' + 2y' + ty = \sin t$, when $y'(0) = 0, y(0) = 1$

   (b) Evaluate $L[t^2 u(t-1) + \delta(t-1)]$.