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B.E. DEGREE END SEMESTER EXAMINATIONS, OCT/NOV 2013

COMMON TO MECHANICAL, MATERIAL SCIENCE AND MANUFACTURING ENGINEERING BRANCH

SIXTH SEMESTER - (REGULATIONS 2008)

ME473/ME9351 FINITE ELEMENT ANALYSIS

Time: 3 Hours Max. Marks: 100

Answer All Questions

PART-A (10 x 2 = 20 marks)

1 What are the advantages of weak formulation?

2 Differentiate between primary and secondary variables with suitable examples

3 Derive the shape functions for a 1D quadratic isoparametric element

4 What are the properties of the Stiffness matrix?

5 Derive the mass matrix for a two noded linear element.

6 What is meant by a CST element? Why is the element so called?

7 With suitable examples and the governing equations distinguish between vector and scalar variable problems.

8 Write the [B] matrix for a CST element.

9 With suitable examples explain what serendipity elements are.

10 Derive the shape functions for linear isoparametric triangular element and plot the variation of the same.

PART-B (5 x 16 = 80 Marks)

11. Determine using any numerical technique, the temperature distribution along a circular fin of length 8cm and radius 1cm. The fin is attached to a boiler whose wall temperature is 120 °C and the free end is insulated. Assume convection coefficient h=10 W/cm^2 °C, Conduction coefficient K= 70 W/cm°C and T_∞ = 40°C. Calculate the temperatures at every 1cm from the left end

12.a Determine the nodal displacements, element stresses and support reactions for the stepped bar loaded as shown in Fig.12.a P_1 = 100 kN and P_2=75 kN. The details of each section of the bar are tabulated below:

Portion	Material	E (GPa)	Area (mm^2)
A	Steel	200	1200
B	Aluminium	70	800

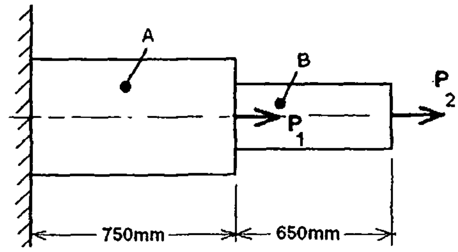


Fig 12.a

OR

12.b Determine the deflection and slope in the beam, loaded as shown in Fig. 12 b, at the mid-span and at the tip. Determine also the reactions at the fixed end.

E_1 = 200 GPa. E_2 = 85GPa, $I = 20 \times 10^{-6}\ m^4$.

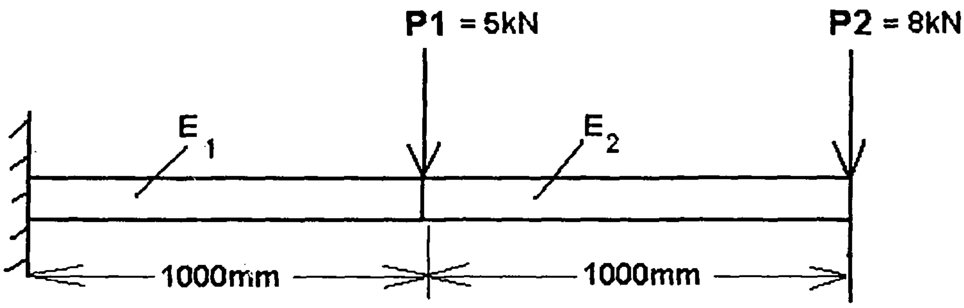


Fig 12.b

13.a Determine the stresses in a square shaft of cross section 1cm x 1 cm as shown in Fig. 13.a subjected to torsion. Considering geometry and boundary condition symmetry 1/8 th of the cross section was modeled using two triangular elements and one bilinear rectangular element as shown. The element matrices are given below. Carry out the assembly and solve for the unknown stress function values.

For Triangle:

$$K = 1/2 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad r = \begin{bmatrix} 29.1 \\ 29.1 \\ 29.1 \end{bmatrix}$$

For Rectangle

$$K = 1/6 \begin{bmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{bmatrix} \qquad r = \begin{bmatrix} 43.6 \\ 43.6 \\ 43.6 \\ 43.6 \end{bmatrix}$$

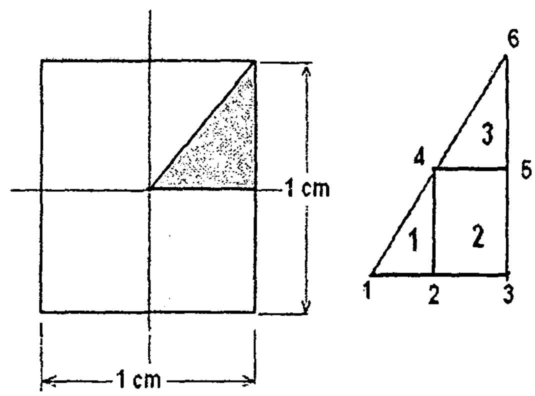


Fig 13.a

OR

13 b i) Determine three points on the 50°C contour line for the rectangular element shown the Fig.13.b. The nodal values are T_1 = 42°C, T_2 = 54°C, T_3 = 56°C, and T_4 = 46°C (8 marks)

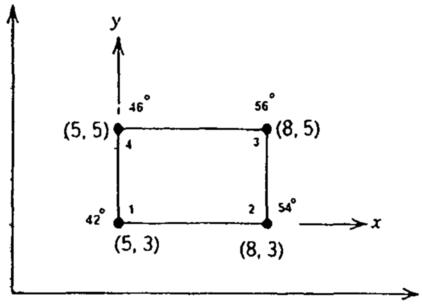


Fig.13.b

ii) Derive the shape functions for one Corner node, one mid-side node and the internal node of a quadratic quadrilateral Lagrangian element (8 marks)

14.a i) A plate of dimensions 15cm x 6 cm x 1cm is subjected to an axial pull of 15kN. Assuming a typical element is of dimensions as shown in the fig.14.a. Determine the strain displacement matrix and constitutive matrix . E = 200 GPa, μ= 0.3. (8 marks)

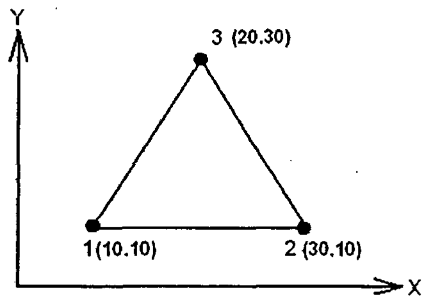


Fig.14.a

ii) Derive the Strain Displacement matrix for a bi linear rectangular element. (8 marks)

OR

14.b i) Derive the constitutive matrix for Plane Stress and Plane strain elements. Give at least two practical examples for Plane Stress and Plane strain analysis. (10 marks)

ii) Using energy approach, derive the stiffness matrix for a ID linear isoparametric element. (6 marks)

15.a i) Using Gauss Quadrature evaluate the following integral using 1 2 and 3 point Integration. Compare with exact solution. $I = \int_{-1}^{+1} (6\xi^3 - 2\xi^2 + 3\xi + 5)d\xi$ (8 marks)

ii) Evaluate the shape functions for a corner node and mid side node of a quadratic triangular serendipity element and plot its variation. (8 marks)

OR

15.b i) What are natural coordinates? What are the advantages of the same? (3 marks)

ii) Explain with suitable examples why we resort to isoparametric transformation. Differentiate between isoparametric, sub parametric and super parametric elements. (5 marks)

iii) For the four noded element shown in Fig15.b determine the Jacobian and evaluate its value at the point (1/2,1/3).

(8marks)

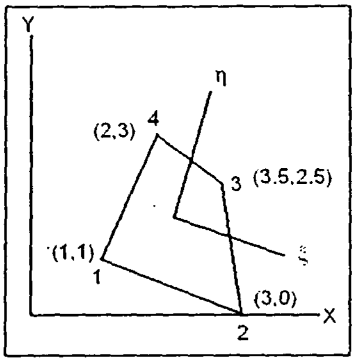


Fig 15.b

$$StiffnessMatrix[K]^e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$\{f\}^e = \frac{ql}{2}\begin{Bmatrix} 1 \\ l/6 \\ 1 \\ -l/6 \end{Bmatrix}$$

$$N_1 = 1-\left(\frac{3x^2}{l^2}\right)+\left(\frac{2x^3}{l^3}\right)$$

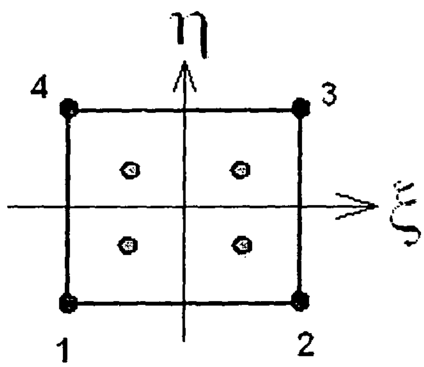
$$N_2 = x-\left(\frac{2x^2}{l}\right)+\left(\frac{x^3}{l^2}\right)$$

$$N_3 = \left(\frac{3x^2}{l^2}\right)-\left(\frac{2x^3}{l^3}\right)$$

$$N_4 = -\left(\frac{x^2}{l}\right)+\left(\frac{x^3}{l^2}\right)$$

$$[M] = \frac{\rho A\ell}{420}\begin{bmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^2 & 13\ell & -3\ell^2 \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 \end{bmatrix}$$

No.of points	Location	Weight W_i
1	$\xi_1 = 0.00000$	2,00000
2	$\xi_1, \xi_2 \pm 0.57735$	1.00000
3	$\xi_1, \xi_3 = \pm 0.77459$ $\xi_2 = 0.00000$	0.55555 0.88888



$$F(\xi,\eta) = \Sigma f\xi_i \eta_i)w_i w_j$$

$$\xi,\eta = 0.57735$$