ANNA UNIVERSITY CHENNAI
B.E./ B.TECH. (FT) DEGREE END SEMESTER EXAMINATIONS – APR / MAY 2012
FOURTH SEMESTER – (REGULATIONS 2004)
BRANCH: ELECTRONICS AND COMMUNICATION ENGINEERING
MA 504 – RANDOM PROCESSES

Time : 3 Hours | **Answer ALL Questions** | **Max. Marks: 100**

Part A **(10 × 2 = 20)**

1. Suppose that 15% of the population of a country are unemployed women and a total of 25% are unemployed. What percent of the unemployed are women?
2. Suppose that 5 good fuses and 2 defective fuses have been mixed up. To find the defective ones, we test them one-by-one, at random and without replacement. What is the probability that we find both of the defective fuses in exactly three tests?
3. Suppose that, on a summer evening, shooting stars are observed at a Poisson rate of one every 12 minutes. What is the probability that 3 shooting stars are observed in 30 minutes?
4. Let X be a Gamma random variable with parameters (r,λ). Find the distribution function of cX, where c is a positive constant.
5. For $\lambda>0$, let $F(x,y)=\begin{cases}1-\lambda e^{-\lambda(x+y)}, & x>0, y>0\\ 0, & otherwise\end{cases}$. Determine if F is the joint probability distribution function of two random variables X and Y.
6. Three points M, N and L are placed on a circle at random and independently. What is the probability that MNL is an acute angle?
7. If the transition probability matrix (TPM) of a Markov chain is $\begin{pmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ and if the initial state probability distribution is (5/6 1/6), find the probability distribution of the chain after 2 steps.
8. A particle moves on a circle through points which have been marked 0,1,2,3,4 (in a clockwise order). At each step it has a probability p of moving to the right and 1-p to the left. Let X_n denote its location on the circle after the nth step. The process $\{X_n, n\geq 0\}$ is a Markov chain. Find the TPM for the chain.
9. Show that $R_{xx}(0)\geq R_{xx}(\tau)$.

10. The auto-correlation of a random process is $R_{xx}(\tau)=3+2e^{-4\tau^2}$. Find the power spectral density of $x(t)$.

Part B **(5 x 16 =80 Marks)**

11.(a)(i) The theaters of a town are showing seven comedies and nine dramas. Ram has seen five of them so far. If the first three movies he has seen are dramas, what is the probability that the last two are comedies? Assume that Ram chooses the shows at random and sees each movie at most once. **(5)**

(ii) There are 3 identical cards that differ only in color. Both sides of one are black, both sides of the second one are red, and one side of the third card is black and its other side is red. These cards are mixed up and one of them is selected at random. If the upper side of this card is red, what is the probability that its other side is black? **(6)**

(iii) Suppose that the loss in a certain investment, in thousands of dollars, is a continuous random variable X that has a density function of the form: $f(x)=\begin{cases} k\left(2x-3x^2\right) & -1<x<0 \\ 0 & otherwise \end{cases}$.

Calculate the value of k. Also find the probability that the loss is at most \$500. **(5)**

12.(a) (i) A certain basketball player makes a foul shot with probability 0.45. Determine for what value of k the probability of k baskets in 10 shots is maximum, and find this maximum probability. **(5)**

(ii) A father asks his sons to cut their backyard lawn. Since he does not specify which of the three sons is to do the job, each boy tosses a coin to determine the odd person, who must then cut the lawn. In the case that all three get heads or tails, they continue tossing until they reach a decision. Let p be the probability of heads and q=1-p, the probability of tails. Find the probability that they reach a decision in less than n tosses; If p=1/2, what is the minimum number of tosses required to reach a decision with probability 0.95? **(6)**

(iii)For a Poisson random variable, obtain the Moment Generating Function and hence find its mean and variance. **(5)**

(OR)

12. (b)(i)Suppose that 90% of the patients with a certain disease can be cured with a certain drug. What is the approximate probability that, of 50 such patients, at least 45 can be cured with the drug? **(5)**

(ii) The profit is \$350 for each computer assembled by a certain person. Suppose that the assembler guarantees his computers for one year and the time between two failures of a computer is exponential with mean 18 months. If it costs the assembler \$40 to repair a failed computer, what is the expected profit per computer? **(6)**

(iii)Obtain the moment generating function of a Gamma Random variable and hence find its mean and variance. **(5)**

13. (a) (i)Let X and Y be positive independent random variables with identical probability density function (pdf) e^{-X} , x>0. Find the joint pdf of U =X+Y and V=X/Y. **(8)**

(ii) Let X be the lifetime of an electronic system and Y be the lifetime of one of its components. Suppose that the electronic system fails if the component does (but not necessarily vice versa). Further suppose that the joint pdf of X and Y (in years) is given by $f(x,y)=\begin{cases}\frac{1}{49}e^{-y/7} & \text{if } 0\le x\le y<\infty\\ 0 & \text{otherwise}\end{cases}$ Determine the expected value of the remaining lifetime of the component when the system dies and also find the covariance of X and Y. **(8)**

(OR)

13.(b)The joint density of (X,Y) is: $f(x,y)=\frac{1}{8}(x+y), 0\le x\le 2, 0\le y\le 2$.Find the correlation coefficient $r(X,Y)$. **(16)**

(OR)

14.(a)(i) Let N(t) be a Poisson process with parameter λ. Determine the coefficient of correlation between N(t) and N(t+τ) for t>0 and τ >0 . **(10)**

(ii) Let $\{X(t), t\ge 0\}$ be a Poisson process with intensity parameter λ. Suppose each arrival is registered with probability p independent of other arrivals. Let $\{Y(t), t\ge 0\}$ be the process of registered arrivals. Prove that $\{Y(t), t\ge 0\}$ is a Poisson process with parameter λp. **(6)**

(OR)

14.(b) Consider a two-state Markov chain with TPM $P=\begin{pmatrix}1-a & a\\ b & 1-b\end{pmatrix}$, 0<a<1, 0<b<1.1.Show that the n–step TPM P^n is given by $P^n=\frac{1}{a+b}\left\{\begin{pmatrix}b & a\\ b & a\end{pmatrix}+(1-a-b)^n\begin{pmatrix}a & -a\\ -b & b\end{pmatrix}\right\}$. 2. Find P^n when $n\to\infty$. **(16)**

15. (a) (i) Two random processes $x(t)$ and $y(t)$ are given by $x(t) = A\cos(wt+\theta)$ and $y(t) = A\sin(wt+\theta)$ where A and w are constants and θ is a uniform random variable over $(0,2\pi)$. Find the cross spectral density function $S_{xy}(w)$ and $S_{yx}(w)$ and verify $S_{xy}(w) = S_{yx}(-w)$ **(10)**

(ii) Show that in an input-output system the energy of a signal is equal to the energy of its spectrum **(6)**

(OR)

15.(b)(i) A wide-sense stationary random process $x(t)$ with autocorrelation function $R_{xx}(\tau) = e^{-a|\tau|}$ with a to be a real positive constant is applied to the input of linear time invariant system with impulse response $h(t) = e^{-bt}u(t)$ where b is a real positive constant. Find the autocorrelation function of the output $y(t)$ of the system. **(8)**

(ii) Let $y(t) = x(t) + W(t)$ where $x(t)$ and $W(t)$ are orthogonal and $W(t)$ is a white noise. Find the autocorrelation function of y(t). **(8)**

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