9. (a) Verify Stokes' theorem for
\[ \vec{F} = (x^2 + y^2)i - 2xyj \]
taken around the rectangle bounded by the lines \( x = \pm a, y = 0 \) and \( y = b \).

(b) Evaluate \( \int_S \vec{F} \cdot dS \), where
\[ \vec{F} = 4xi - 2y^2j + z^2k \]
and \( S \) is the surface bounding the region \( x^2 + y^2 = 4, z = 0 \) and \( z = 3 \).

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B.Tech 2nd Semester Exam., 2016

MATHEMATICS—II

Time: 3 hours

Full Marks: 70

Instructions:
(i) The marks are indicated in the right-hand margin.
(ii) There are NINE questions in this paper.
(iii) Attempt FIVE questions in all.
(iv) Question No. 1 is compulsory.

1. Answer any seven of the following as directed: \( 2 \times 7 = 14 \)

(a) The series
\[ \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots \]
converges, if

(i) \( p > 0 \)

(ii) \( p < 1 \)

(iii) \( p > 1 \)

(iv) \( p \leq 1 \)

(Choose the correct option)
(b) The series
\[ 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots \]
is
(i) oscillatory
(ii) conditionally convergent
(iii) divergent
(iv) absolutely convergent
(Choose the correct option)

c) The period of \(|\sin t|\) is \(\bigcirc\).
(Fill in the blank)

d) In the Fourier series expansion of \(f(x) = |\sin x|\)
in \((-\pi, \pi)\), the value of \(b_n = \bigcirc\).
(Fill in the blank)

e) The function
\[ f(x) = \begin{cases} 1-x, & \text{in } -\pi < x < 0 \\ 1+x, & \text{in } 0 < x < \pi \end{cases} \]
is an odd function.
(Write True or False)

(f) \(\int_0^{\pi/2} \int_0^a \sin \theta \ r \ d\theta \ dr = \frac{a^2}{8}\).
(Fill in the blank)

(g) \(\int_0^{\pi/2} \int_0^a \sin \theta \ r \ d\theta \ dr\)

(h) On changing to polar coordinates
\[ \int_0^{2a} \sqrt{(2ax-x^2)} \ dx \ dy \]
becomes \(\bigcirc\).
(Fill in the blank)

(i) If \(\nabla \cdot \vec{F} = 0\), then \(\vec{F}\) is called \(\bigcirc\).
(Fill in the blank)

(j) If \(\vec{A}\) is such that \(\nabla \times \vec{A} = 0\), then \(\vec{A}\) is called \(\bigcirc\).
(Fill in the blank)

(2) Discuss the convergence of the series
\[ x + \frac{2^2 \cdot x^2}{2^3} + \frac{3^3 \cdot x^3}{3^4} + \frac{4^4 \cdot x^4}{4^5} + \frac{5^5 \cdot x^5}{5^6} + \ldots \infty \]
\((x > 0)\)

(b) Prove that the series
\[ \frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \ldots \]
converges absolutely.
3. (a) Expand \( f(x) = x \sin x \) as a Fourier series in the interval \( 0 < x < 2\pi \).

(b) Obtain Fourier expansion for the function
\[
f(x) = \begin{cases} 
\pi + x, & \text{if } -\pi \leq x \leq 0 \\
\pi - x, & \text{if } 0 \leq x \leq \pi 
\end{cases}
\]
and \( f(x + 2\pi) = f(x) \).

4. (a) Find half-range cosine series for the function \( f(x) = x^2 \), \( 0 < x < 2 \).

(b) Find the Laplace transform of \( te^{-t} \sin 3t \).

5. (a) Find the value of
\[
L(\int_0^t e^{-t} \sin t \, dt)
\]

(b) Find the inverse Laplace transform of
\[
\frac{s + 2}{s^2(s + 1)(s - 2)}
\]

6. (a) By changing the order of integration in
\[
I = \int_0^1 \int_{y^2}^{2-y} xy \, dx \, dy
\]
evaluate the integral.

(b) Evaluate
\[
\iint r \sin \theta \, dr \, d\theta
\]
over the cardioid \( r = a(1 - \cos \theta) \) above the initial line.

7. (a) Evaluate
\[
\iint_R (x + y)^2 \, dx \, dy
\]
where \( R \) is the parallelogram in the \( xy \)-plane with vertices \((1, 0), (3, 1), (2, 2), (0, 1)\) using the transformation \( u = x + y \) and \( v = x - 2y \).

(b) Find by triple integration, the volume of the sphere \( x^2 + y^2 + z^2 = a^2 \).

8. (a) Show that
\[
\sum_{n=0}^{\infty} n(n + 1)r^{n-2} = 2(r^n + 1)r^n - 2
\]

(b) Find the work done in moving a particle in the force field
\[
\vec{F} = 3x^2 \hat{i} + (2xz - y) \hat{j} + zk
\]
along the curve defined by \( x^2 = 4y \), \( 3x^3 = 8z \) from \( x = 0 \) to \( x = 2 \).