Code : 211202

B.Tech 2nd Semester Exam., 2015

MATHEMATICS—II

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Choose the correct or best alternatives (any *seven*) : 2×7=14

(a) In a series of positive terms Σu_n, if

$$\lim_{n \to \infty} u_n \neq 0$$

then the series Σu_n is

(i) convergent

(ii) divergent

(iii) not convergent

(iv) oscillatory

(b) The p-series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots$ to ∞ is divergent for

(i) $p=1$ *(ii)* $p>1$

(iii) $p<1$ *(iv)* $p \le 1$

(c) The series $a-a+a-a+a-\cdots$ to ∞ is

(i) convergent

(ii) divergent

(iii) oscillatory

(iv) None of the above

(d) The Laplace transform of a signal $y(t)$ is

$$y(s)=\frac{1}{s(s-1)},$$

then its final value is

(i) -1

(ii) 0

(iii) 1

(iv) unbounded

(e) The Laplace transform of the function $f(t)=t$, starting at $t=a$ is

(i) $\frac{1}{(s+a)^2}$ *(ii)* $\frac{e^{-as}}{(s+a)^2}$

(iii) $\frac{e^{-as}}{s^2}$ *(iv)* $\frac{a}{s^2}$

(f) If from the function $f(t)$ one forms the function $\psi(t) = f(t) + f(-t)$, then $\psi(t)$ is

(i) even

(ii) odd

(iii) neither even nor odd

(iv) both even and odd

(g) The triple integral $\iiint_T dx\,dy\,dz$ gives

(i) volume of region T

(ii) surface area of region T

(iii) area of region T

(iv) density of region T

(h) The double integral

$$\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$$

is

(i) 0

(ii) π

(iii) $\pi/2$

(iv) 2

(i) $\nabla \times (\nabla \times \vec{A})$, where $\vec{A}$ is a vector, is equal to

(i) $\vec{A} \times \nabla \times \vec{A} - \nabla^2 \vec{A}$

(ii) $\nabla^2 \vec{A} + \nabla(\nabla \cdot \vec{A})$

(iii) $\nabla^2 \vec{A} + \nabla \times \vec{A}$

(iv) $\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

(j) If $(\vec{A} \times \vec{B}) \times \vec{C} = \vec{A} \times (\vec{B} \times \vec{C})$, then

(i) $\vec{A}$, $\vec{B}$ are collinear

(ii) $\vec{A}$, $\vec{B}$ are perpendicular

(iii) $\vec{A}$, $\vec{C}$ are collinear

(iv) $\vec{A}$, $\vec{C}$ are perpendicular

2. *(a)* Test for convergence the series whose nth term is $n^{\log x}$.

(b) Test the series for convergence

$$1 + \frac{3}{7}x + \frac{3\cdot 6}{7\cdot 10}x^2 + \frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3 + \frac{3\cdot 6\cdot 9\cdot 12}{7\cdot 10\cdot 13\cdot 16}x^4 + \cdots \text{ to } \infty$$ 14

3. (a) Define absolutely and conditionally convergent series. Show that the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots \text{ to } \infty$$

is convergent but not absolutely convergent.

(b) Find the Fourier series expansion of the periodic function of period 2π

$$f(x)=x^2, \quad -\pi \le x \le \pi$$

Hence find the sum of the series

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$$ 14

4. (a) Find the Fourier series for $f(x)$, where

$$f(x)=\begin{cases}-\pi, & -\pi < x < 0\\ x, & 0 < x < \pi\end{cases}$$

Hence deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$$

(b) Find the Fourier half-range even expansion of the function

$$f(x)=-\frac{x}{e}+1, \; 0 \le x \le e$$ 14

5. (a) Find the Laplace transform of—

(i) $t^2 \cos at$;

(ii) $e^{-4t}\dfrac{\sin 3t}{t}$.

(b) Find the Laplace transform of $\dfrac{1-\cos t}{t^2}$. 14

6. (a) Show that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3}\,dy \ne \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3}\,dx$$

(b) Evaluate $\iint r^4 \cos^3\theta \, dr\, d\theta$ over the interior of the circle $r=2a\cos\theta$. 14

7. (a) Evaluate :

$$\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dx\, dy\, dz$$

(b) Find the length of the arc of the curve $y=e^x$ from the point (0, 1) to (1, e). 14

8. (a) Find the directional derivatives of $\phi = xy^2 + yz^2$ at the point (2, −1, 1) in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$.

(b) Find the magnitude of the velocity and acceleration of a particle which moves along the curve $x=2\sin 3t$, $y=2\cos 3t$, $z=8t$ at any time $t>0$. Find unit tangent vector to the curve. 14

(Continued)

9. *(a)* If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that

$$\operatorname{div}(\operatorname{grad} r^n) = n(n+1)r^{n-2}$$

(b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where

$$\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$$

and C is the rectangle in the xy-plane bounded by $y = 0$, $x = a$, $y = b$, $x = 0$. 14

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