1. (a) Using Gauss-Jordan method, find the inverse of the matrix $A$

$$
A = \begin{bmatrix}
1 & 1 & 3 \\
1 & 3 & -3 \\
-2 & -4 & -4
\end{bmatrix}.
$$

(b) Determine the value of $\lambda$ for which the following set of equations may possess non-trivial solution

$$
3x + y - \lambda z = 0,
4x - 2y - 3z = 0,
2\lambda x + 4\lambda + \lambda z = 0
$$

for each permissible value of $\lambda$, determine the general solution.

2. (a) Find the eigen values and eigen vectors of the matrix

$$
A = \begin{bmatrix}
2 & 1 & -1 \\
1 & 1 & -2 \\
-1 & -2 & 1
\end{bmatrix}.
$$

(b) Prove that every Hermitian matrix can be written as $A \in iB$, where $A$ is real and symmetric and $B$ is real and skew-symmetric.

UNIT—II

3. (a) Solve:

$$(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0.$$
(b) Find the orthogonal trajectories of a system of parabola \( y^2 = 4a(x + a) \).

(c) Find the complete solution of
\[ (D^4 + 4D - 3)y = X e^{3x}. \]

4. (a) Using the method of variation of parameter, solve:
\[ \frac{d^2y}{dx^2} + a^2 y = \sec ax. \]

(b) An e.m.f. \( E \sin pt \) is applied at \( t = 0 \) to a circuit consisting of a capacitance \( C \) and inductance \( L \). The current \( i \) satisfies the equation
\[ L \frac{di}{dt} + \frac{1}{C} \int_0^t i \, dt = E \sin pt, \]
if \( p^2 = \frac{1}{LC} \) and initially the current \( i \) and charge \( q \) are zero. Show that the current at time \( t \) is
\[ \left( \frac{Et}{2L} \right) \sin pt, \]
where \( \frac{dq}{dt} = i \).

UNIT—III

5. (a) Evaluate:
\[ L \left\{ \int_0^t \frac{\sin t}{t} \, dt \right\}. \]

(b) Evaluate:
\[ L^{-1} \left\{ \frac{s^2 + s}{(s^2 + 1)(s^2 + 2s + 2)} \right\}. \]

(c) Use Convolution theorem to evaluate:
\[ L^{-1} \left( \frac{1}{s^3(s^2 + 1)} \right). \]

6. (a) Using Laplace transform method, solve
\[ ty'' + 2y' + ty = \sin t, \text{ when } y(0) = 1. \]
(b) In an electrical circuit with e.m.f. $E(t)$ resistance $R$ and inductance $L$, the current $i$ build up at the rate given by

$$L \frac{di}{dt} + Ri = E(t).$$

If the switch is connected at $t = 0$ and disconnected at $t = a$, find the current $i$ at any instant.

UNIT—IV

7. (a) Solve the partial differential equation

$$(z^2 - 2yz - y^2) p + (xy + zx)q = xy - zx.$$ 

(b) Solve:

$$2z + p^2 + qy + 2y^2 = 0.$$ 

8. (a) Solve:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x.$$ 

(b) Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to $u(0, t) = u(l, t) = u(x, 0) = 0$ and $u(x, a) = \sin \frac{n \pi x}{l}$.