BT-5/D-13
AUTOMATA THEORY
Paper-CSE-305 Opt. (II)

Time allowed : 3 hours] [Maximum marks : 100

Note : Attempt any five questions.

1. Give deterministic finite automata for the following language over alphabet {0, 1}
   (i) Strings starting with a leading 0 and not containing consecutive 1s.
   (ii) Strings with even number of 0s and odd number of 1s.
   (iii) Strings containing third symbol from the right as 1.
   (iv) All strings that have exactly one double letter in them.

2. (a) Prove that
   (i) \((00^* \cdot 1)^* \cdot 1 = 1 + 0 (0 + 10)^* \cdot 11\)
   (ii) \(((111)^*)^* = (11 + 111)^*\)

   (b) Consider the two regular expressions
   \(r_1 = 0^* + 1^*\) \(r_2 = 01^* + 10^* + 10 + (01)^*\)
   (i) Find a string corresponding to \(r_1\) but not to \(r_2\).
   (ii) Find a string corresponding to both \(r_1\) and \(r_2\).
   (iii) Find a string in \{(0, 1)^*\} corresponding to neither \(r_1\) nor \(r_2\).
   (iv) Find a string corresponding to \(r_2\), but not to \(r_1\).

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3. (a) Design a Moore’s machine which will count how many times substring aab occurs in a long input string. Count can be maintained by printing 1 each time aab occurs.

(b) Define and explain Mealy machine.

4. (a) Construct a finite automation accepting all strings over \( \{0,1\} \) ending in 010 or 0010.

(b) Find all strings of length 5 or less in the regular set represented by

(i) \((ab + a)^* (aa + b)\)

(ii) \((a*b + b*a)^* a\)

(iii) \(a^* + (ab + a)^*\)

5. (a) \(L = \{0^n1^n \mid n = 0, 1, 2, \ldots\}\) Prove that \(L\) is not a regular language and write CFG to generate \(L\).

(b) \(L = \{w \mid w \in \{0,1\}^*\}\). Write CFG to generate \(L\) where \(w\) consists of equal number of 0’s and 1’s.

6. (a) Design a PDA to recognize all words in \(\{a^n b^n \mid n \geq 0\}\).

(b) Show that the set of all strings over \(\{a, b\}\) consisting of equal number of \(a\)'s and \(b\)'s is accepted by a deterministic PDA.

7. (a) Design a Turing Machine to recognize an arbitrary string divisible by 4 from \(\Sigma = \{0,1,2\}\)

(b) Define and explain Universal Turing Machine.

8. (a) Show that Fibonacci numbers are generated by a primitive recursive function.

(b) Prove that "There is a recursive language \(L\) over \(\{a,b\}\) such that \(L - \{\Lambda\}\) is not context-sensitive."

(c) Define and explain unrestricted grammar.