8. (a) Construct the following graphs:
(i) Eulerian but not Hamiltonian.
(ii) Hamiltonian but not Eulerian.
(iii) Neither Eulerian nor Hamiltonian.
(iv) Eulerian and Hamiltonian.
(b) Define: Graph, Simple Graph, Pseudo graph and Weighted graph.
(c) A tree of order $n$ has size $(n-1)$. Prove.

SECTION-I

1. (a) If $A$, $B$ and $C$ be subsets of the universal set $U$, then prove:

(i) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(b) In a city 60% of the residents can speak German and 50% can speak French. What percentage of residents can speak both the languages, if 20% residents can not speak any of these languages?

2. (a) (i) If $R$ be an equivalence relation defined on a non-empty set $A$ and $x, y$ be arbitrary elements in $A$, and $x \in [x]$ and $y \in [x]$, then $[x] = [y]$.
(ii) Prove by method of induction:

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$
(b) Let \( f : A \rightarrow B \) and \( g : B \rightarrow C \) be two functions, then \((gof)\) is one-one if both \( f \) and \( g \) are one-one and \((gof)\) is onto if both \( f \) and \( g \) are onto.

(ii) Give an example of a function which is

(a) Injective but not Surjective

(b) Bijective

(c) Surjective but not Injective,

(d) Constant.

SECTION-II

3. (a) Prove by constructing truth table

\[ P \rightarrow (Q \lor R) = (P \rightarrow Q) \lor (P \rightarrow R). \]

(b) Solve the recurrence relation \( S_n - 7S_{n-1} + 10S_{n-2} = 0 \), \( S_0 = 0 \) and \( S_1 = 3 \) by using generating function where, \( n \geq 2 \).

(c) Find the total solution of the difference equation

\[ S_n - S_{n-1} = 5, \text{ given that } S_0 = 2. \]

4. (a) Solve the difference equation

\[ \sqrt{S_{n-1}} + \sqrt{S_{n-2}} + \sqrt{S_{n-3}} + \cdots, \text{ given that } S_0 = 4. \]

(b) Find the total distinct numbers of six digits that can be formed with 0, 1, 3, 5, 7 and 9 and how many of them is divisible by 10?

(c) Discuss the importance of recurrence relations in the binary algorithm.

SECTION-III

5. (a) If \( G \) is a set of Real numbers (non-zero) and let

\[ a \ast b = \frac{ab}{2}, \text{ show that } (G, \ast) \text{ is an abelian group.} \]

SECTION-IV

7. (a) Determine whether the graph given below by its adjacency matrix is connected or not, where the matrix

\[ A = \begin{pmatrix}
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0
\end{pmatrix}. \]

(b) Find the complement of the following graphs:

(c) If \( T \) is a binary tree of height \( h \) and order \( p \), then \( (h + 1) \leq p \leq 2^{(h + 1)} - 1. \)