UNIT-I

1. (a) Let A, B, C be arbitrary sets, prove that:
   (i) \((A - B) - C = (A - C) - B\).
   (ii) \((A - B) - C = (A - C) - (B - C)\).
   (iii) \((A \cap B) \subseteq B \subseteq (A \cup B)\).
   (iv) \((A \cap B) \subseteq A \subseteq (A \cup B)\).

   (b) Show that if any four numbers are chosen from 1 to 6, then two of them will add to 7.

2. (a) Define the following:
   (i) Reflexive relation.
   (ii) Anti-symmetric relation.
   (iii) Transitive relation.
   (iv) Irreflexive relation.

   (b) Let \(P(S)\) be the power set of the set \(S = \{1, 2, 3\}\). Deduce the relation set and construct the hasse diagram using lattice \((P(S), \cup, \cap)\).

   (c) Find the greatest and smallest elements of the lattice of Question No. 2(b).
UNIT-II

3. (a) Construct the truth tables for the following statements:
   (i) \( p \rightarrow p \).
   (ii) \( (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)) \).

(b) Determine the number of ways to place \((2t + 1)\) distinguishable balls in three distinct boxes so that any two boxes together will contain more balls than the other.

4. (a) Solve the difference equation:
   \[ a_n - 7a_{n-1} + 10a_{n-2} = 0 \]
   satisfying the conditions,
   \[ a_0 = 0 \text{ and } a_1 = 6. \]

(b) John made the following statements:
   (i) I love Lucky.
   (ii) If I love Lucky, then I also love Vivian.

Given that John either told the truth or lied in both cases, determine whether John really loves Lucky.

UNIT-III

5. (a) For any arbitrary group \( G \), prove:
   (i) identity element is unique.
   (ii) inverse of an element is unique.
   (iii) \( (a^{-1})^{-1} = a \).
   (iv) \( (ab)^{-1} = b^{-1}a^{-1} \).

(b) Let \( G \) be a finite group and \( H \) be a subgroup of \( G \). If \( aH = \{ah : h \in H\} \), show that for any \( a, b \in G \), either \( aH = bH \) or \( aH \cap bH = \phi \).
6. (a) Is addition (+) on the set \( s = \{0, 1, -1\} \) an operation. Why?
(b) State and prove Lagrange's Theorem.
(c) Prove that \( Z_p \) is a field, where \( p \) is a prime number.

UNIT-IV

7. (a) Find the shortest path from \( a \) to \( e \) in the following weighted graph using Dijkstra's algorithm:

(b) Prove that there is one and only one path from root to any node in a tree.

8. (a) Construct the binary tree for the expression

\[ E = (a + b)^* \left( \left( d - e \right) / \left( f - g \right) \right) - h/k \]

and find its preorder and postorder traversals.

(b) Prove that \( K_{3,3} \) is non-planar.

(c) Check and state which of the following are bi-partite graphs and why?

(i)  
(ii)  
(iii) \( (8,6,6) \)